System Identification Via Auto-Encoders: A Comparison Between Data-Driven and Physics-Informed Solutions

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ABSTRACT

Variational Auto-Encoders (VAE) and Long Short-Term Memory (LSTM) are investigated in this article in the framework of Structural Health Monitoring (SHM). The presented approaches aims to combine sensor data with numerical modelling of the dynamical system in a Reduced Order Modelling setting. Two Finite Element case-of-study are proposed with the scope of identify non-linear forces and physical parameter degradation. Starting from a reduced set of sensors data and different levels of knowledge of the physical system, the reconstruction capabilities of time series data are presented and compared for both proposed architectures.

INTRODUCTION

With the advances in the development of predictive Machine Learning algorithms, physics-informed solutions have attracted great attention in the field of Structural Health Monitoring (SHM), by combining sensor data with numerical modelling. Different approaches may be adopted, ranging from white-box modelling to black-box modelling, depending on data availability. Grey-box modelling considers a partial level of knowledge of the physical system embedded in the network architecture. This setting has several advantages since often it is not possible to reach a perfect knowledge of the system e.g. unknown non-linearities, un-predictable physical phenomena. However, the effort needed in reaching the right level of accuracy of analytical or Finite Element (FE) modelling might limit the usage of these techniques compared to standard black-box models. Moreover, FE models of large-scale structures are computationally expensive and require data-reduction approaches, i.e. Reduce Order Modelling (ROM). The autoencoders (AEs) have been widely used in literature to approach ROM in the field of struc-

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tural mechanics [1,2]. Long-Short-Term Memory (LSTM) combined with AE can reach fast and accurate reconstruction and prediction of the operational life of a structure from the knowledge of a limited set of sensor data. Nevertheless, these models do not take into account the physics of the system representing a purely data-driven approach. The recent development of Physics-Informed (PI) Neural ODE in a Variational autoencoder (VAE) [3] framework gives some advantages over the data-driven solution of LSTM-AE. In this contribution, LSTM-AE and physics-informed VAE are compared in terms of the reconstruction capability of the dynamics of the physical system, by addressing the identification of i) internal non-linear forces and ii) physical parameter degradation. Two FE case-of-study are proposed: i) a 2D cantilever beam with grounded cubic spring, ii) 3D model of Double-Cantilever-Beam (DCB). The global parametric reduction basis is computed through Proper Orthogonal Decomposition (POD) [4] and embedded in the VAE architecture.

GENERAL REDUCED ORDER FINITE ELEMENT MODELLING WITH AUTO-ENCODERS

The numerical simulation of a large-scale component involves the usage of FE model with a large amount of Degrees of Freedom (DoFs). In order to reduce the computational power needed to simulate these models, a proper Model Order Reduction (MOR) technique is used to perform the projection of the Full Order Model (FOM) with $\mathbf{x} \in \mathbb{R}^{n_f}$ DoFs into a lower dimensional space of $\mathbf{q} \in \mathbb{R}^{n_r}$ DoFs with $n_r \ll n_f$. Given the reduction basis $\Phi \in \mathbb{R}^{n_f \times n_r}$ the following approximation holds:

$$\mathbf{x} \cong \Phi \mathbf{q} \tag{1}$$

The state-space formulation of the reduced model can be written as follows:

$$\begin{cases} \dot{\mathbf{z}} = f(\mathbf{z}, \mathbf{u}) \\ \mathbf{y} = \mathbf{C_r}(\mathbf{z}) + \mathbf{D}\mathbf{u} \end{cases} \tag{2}$$

where $\mathbf{z} \in \mathbb{R}^{2n_r}$ is the vector of concatenated generalized coordinates and time derivatives $\dot{\mathbf{q}}$. By solving Eq.2, $\mathbf{z}(t)$ is projected back into the FOM to obtain $\mathbf{x}(t)$. This transformation goes from low-dimensional to high-dimensional space and in a ML framework can be interpreted as a decoding operation achieved by the decoder module on the network $\mathscr{D}(\mathbf{z})$, as indicated in Figure 1, where $\mathbf{C_r}(\mathbf{z}) = \mathbf{C}\mathscr{D}(\mathbf{z})$. The encoder $\mathscr{E}(x)$ performs the opposite operation of the decoder by mapping the FOM into the ROM as in Eq.1. $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ in Figure 1 are respectively estimated DoFs and measurements. With reference to Figure 1, the decoder $\mathscr{D}(z)$ can be modelled in two ways:

- $\mathscr{D}(z) = \Phi$: neural network with a single layer with fixed weigths and zero bias. The reduction basis Φ is pre-computed.
- $\mathscr{D}(z) = NN(z)$: multi-layer perceptron with trainable parameters

In the second case, the reduction basis is trainable and defines a non-linear transformation but lacks of physical constrains. The AE model shown in Figure 1 can be used for faster simulation of high order FE systems when the input data is the FOM vector of

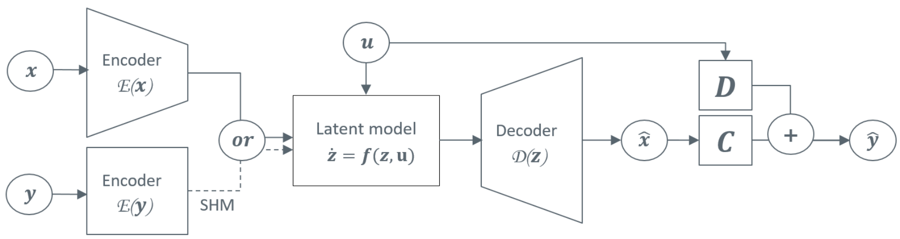

Figure 1. Scheme of Reduced Order Modelling simulation with Autoencoders.

DoFs $\mathbf{x}$. The scope of this article is to use AEs for SHM purposes. During the operational life of a structure, a subset of sensors are measured, thus the full time histories of the FOM are unknown. From here, the need to replace the encoder with $\mathscr{E} = \mathscr{E}(y)$, which takes as input the measurements $\mathbf{y} \in \mathbb{R}^{n_y}$ and returns the initial time conditions of the reduced system. Given the architectures shown in Figure 1, two models are investigated in this contribution:

Variational Autoencoder (VAE): the encoder $\mathscr{E}(y)$ is composed by a multilayer perceptron (MLP) and a recurrent neural network (RNN) to estimate the initial state $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$ respectively. More details can be found in [3]. The model $f(\mathbf{z}, \mathbf{u})$ is assumed to be partially known, i.e. $f(\mathbf{z}, \mathbf{u}) = f_{baseline}(\mathbf{z}, \mathbf{u}) + NN(\mathbf{z})$. The baseline function indicates the linear system, while the residual term takes into account for unknown phenomena and it is modelled as a trainable MLP. The decoder $\mathscr{D}(z)$ can be trainable or non-trainable.

Long Short-Term Memory (LSTM): the encoder $\mathscr{E}(y)$ is composed by 2 LSTM [5] layers to compress the original input into a lower-dimensional representation. The decoder path $\mathscr{D}(z)$ is a mirror of the encoder and aims to reconstruct the original input. In this latter case $f(\mathbf{z}, \mathbf{u})$ is a black-box model.

NON-LINEAR FORCE IDENTIFICATION: CANTILEVER BEAM

A cantilever Euler-Bernoulli beam is investigated in this section with the aim to identify non-linear forces generated by the presence of a cubic grounded spring attached to the tip (Figure 2). This benchmark has been presented and studied in several contributions [6] and explored in [7] for the shape reconstruction through VAE and LSTM. The beam model is described in [7] and is discretized via 2D FE analysis with $N_e = 15$ beam elements for a total number of DoFs $n_d = 2N_e$, i.e. vertical displacement z and rotation along $y-$axis. The system has been simulated via Euler implicit integrator in a time window of $T = 100s$. The parameter k_n, associated to the non-linear spring, is sampled in order to create the dataset for different values of non-linearities. The value of k_n ranges between 0.5% and 1.0% of the traslational stiffness associated to the end node of the beam and 5 uniformly distributed samples are selected within the range. Differently from [7], the free-dynamic of the beam, i.e. $f_{ext}(t) = 0$, is simulated by imposing the first mode shape of the linear beam as initial conditions. The global reduction basis Φ is computed by applying POD on the snapshots data of the FOM solution. A total of 5 meaningful modes, i.e. $n_r = 5$ is selected from SVD analysis. A subset of $n_y = 5$ measurements are selected by ensuring the system observability.

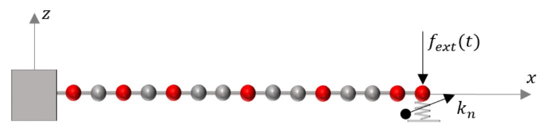

Figure 2. FE model of a clamp-free beam with a grounded spring.

Training scenario: Among the 5 simulated samples, 4 of these are used for the training. The sample in the middle of the range of k_n is used as validation set. In order to augment the size of the training dataset, a windowing approach with a pre-defined length w_l is applied on the time-series data. In this way, the encoder can learn all the possible combination of initial conditions of the latent model. A parameter $o_w \in [0, 1]$ is defined to tune the overlap between each window: $o_w = 1$ indicates 100% of overlap which means that each sequence is shifted about 1 time step; $o_w = 0$ indicates zero-overlap which means that each window is sequential to the previous one. For all the experiments, a learning rate of $1e^{-}3$ and batch size equal to 32 are used.

Encoder: in the case of VAE, we tested different combinations by disabling the encoder and by supposing to know the initial conditions of the latent model from the simulated data. This test allows to focus on the estimation of the nonlinear term of the system which can be identified in two scenarios as follows in the next point.

Enable Reduction Forces (ERF): in the case of VAE, the model is assumed to be partially known. The nonlinear forces in a FOM formulation is identified as a function of $\mathbf{x}$, i.e. $f(\mathbf{x})$. The equations of motion projected on the reduced model will read:

$$\dot{\mathbf{z}} = f_{baseline}(\mathbf{z}, \mathbf{u}) + f(\mathbf{x}) = f_{baseline}(\mathbf{z}, \mathbf{u}) + \Phi^T f(\Phi \mathbf{z}) \quad (3)$$

The computation of the last term becomes expensive for large systems since it requires to perform the projection two times at each time step. Hyper-reduction techniques are available in literature to address this issue but their implementation in the VAE are out of the scope of the presented article. In a ML perspective, the term $\Phi^T f(\Phi \mathbf{z})$ can be approximated with a general function $\tilde{f}(z)$ acting on the reduced system. In the following, if ERF is specified, the full projection is performed. Otherwise the approximated formulation is considered.

The tests performed through VAE and LSTM are listed in Table I and compared in Figure 3, 4 and 5. For all the experiments, the nonlinearities are shown as summation of forces over all DoFs expressed in the reduced space, i.e. $\sum_{i=1}^{n_r} f(z)$, where the expression of $f(z)$ changes accordingly with ERF parameter. MSE and MAE indicate respectively the Mean Square Error and the Mean Absolute Error averaged on all the DoFs. The following concluding remarks can be made from the presented experiments:

Comparison between case 1 and 2: the estimation of forces when ERF is set to True results to be more accurate. However, no significant improvements are observed for this use case and the accuracy of the approximation with $\tilde{f}(z)$ should be evaluated on a more complex case for which more iterations might be needed.

Comparison between case 2 and 3: the POD reduction basis gives more accurate reconstruction with respect to a trainable decoder. The lack of accuracy obtained through

TABLE I. PERFORMED EXPERIMENTS. D: FULL-FIELD DISPLACEMENTS. NL: NONLINEARITIES.

	Model	Encoder	ERF	Decoder	o_w	w_l	Trainable param.	Estimation	MSE	MAE
1	VAE	Disabled	False	Φ	0.25	400	38297	D + NL	$1.4e^{-4}$	$6.9e^{-3}$
2	VAE	Disabled	True	Φ	0.25	400	44082	D + NL	$2.6e^{-5}$	$3.2e^{-3}$
3	VAE	Disabled	False	MLP	0.25	400	43531	D + NL	$1.0e^{-4}$	$7.2e^{-3}$
4	VAE	Enabled	False	Φ	0	200	31641	D + NL	$6.0e^{-4}$	$1.5e^{-2}$
5	VAE	Enabled	False	Φ	1	200	31641	D + NL	$2.0e^{-4}$	$8.0e^{-3}$
6	LSTM-AE	Enabled	—	MLP	1	200	2281030	D	$4.0e^{-6}$	$4.0e^{-4}$
7	LSTM-AE	Enabled	—	MLP	1	200	20230	D	$7.0e^{-5}$	$6.0e^{-3}$

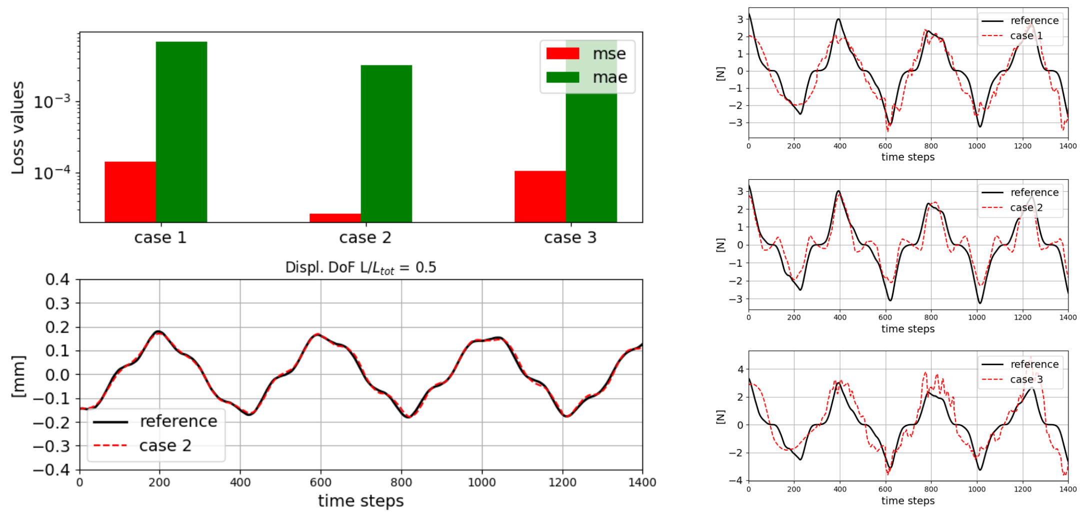

Figure 3. Case 1-3 comparison. Top left: MAE and MSE averaged on all the DoFs. Bottom left: reconstruction of un-measured sensor. Right: estimation of nonlinearities.

MLP might be explained in the lack of physical constraints when the decoder is trainable.

Comparison between case 4 and 5: The architectures compared here include a trainable encoder as shown in the figure (1) for estimating initial conditions. The difference between the two cases consists of the overlap parameter o_w used for sequences equal to 0 and 1 for case 4 and case 5, respectively. From the loss on validation and graphical trends an overlap equal to one guarantees better generalization performance.

Comparison between case 5 and 6: In the last comparison, the VAE architecture and an LSTM-AE are compared. The latter model shows a lower loss value in validation but it is unable to estimate the contribution of nonlinearities. Both the configuration with a larger and a similar number of parameters (case 6 and case 7) performed better but in the latter case the validation loss are closer.

PARAMETER ESTIMATION: COMPOSITE DOUBLE CANTILEVER BEAM

In this section, the problem of physical parameter estimation is addressed through VAE methodology. The model investigated is a Double Cantilever Beam (DCB) specimen shown in Figure 6 which is used to study the delamination properties of composite

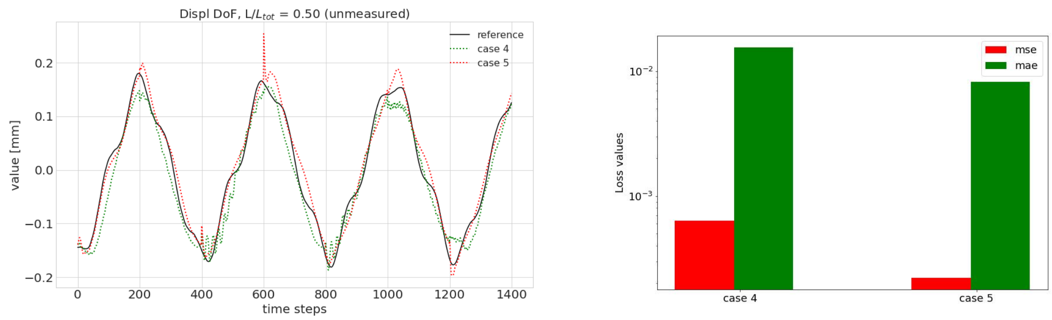

Figure 4. Reconstruction of un-measured sensor (left) and MAE and MSE averaged on all the DoFs (right).

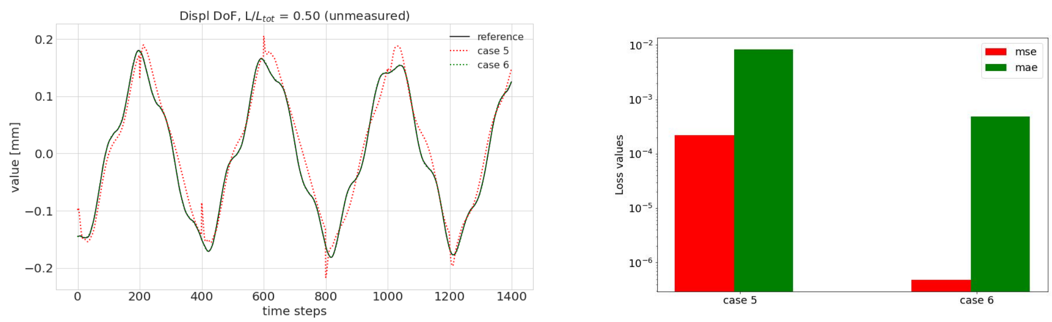

Figure 5. Reconstruction of un-measured sensor (left) and MAE and MSE averaged on all the DoFs (rigth).

laminates. The DCB is tested by applying tip displacements in opposite directions such that the initial crack can propagate. A non-linear behavior is observed in Figure 6: the model behaves linearly until the crack starts the propagation and the global stiffness of the model decreases. A simplified way to model the DCB without recurring to non-linear FE elements i.e. cohesive elements, is shown in Figure 7: node-to-node spring elements are distributed between the two laminae [8] and the stiffness of the springs gradually decreases over time. In this case of study, the Parametric MOR (PMOR) is adopted for the computation of the reduction basis Φ. The method is the one presented in [9]: the stiffness of the non-linear FE model is sampled and a linear model can be defined for each sample of stiffness value. In this case, each sample corresponds to a different stage of degradation of the interface composed by springs. The reduction basis Φ is computed by performing SVD analysis on the concatenated local basis and each local basis is defined by computing the eigen-value problem. A scheme of the described workflow is shown in Figure 8. A total of 18 meaningful modes, i.e. $n_r = 18$ and a subset of $n_y = 12$ measurements equally distributed along the lamina length are selected. In the following, the training scenario and the experiment performed are explained.

Training scenario: The descending curve of Figure 8 has been sampled with 18 data

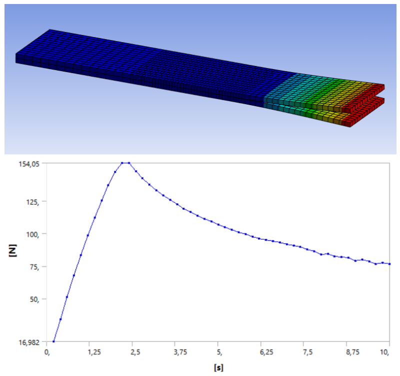

Figure 6. FE model of the DCB and tip reaction force.

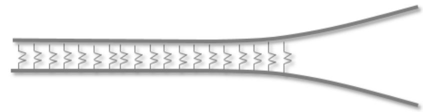

Figure 7. Simplified DCB model with note-to-node springs at the interface.

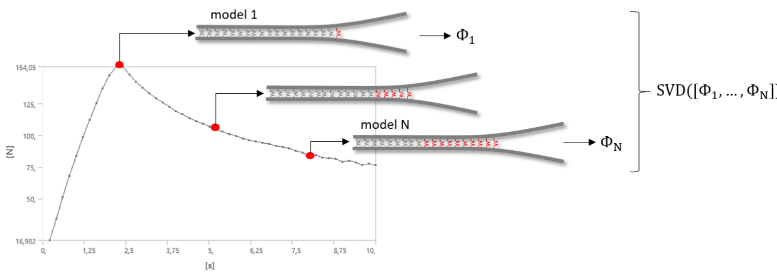

Figure 8. Workflow for the computation of the global PMOR reduction basis.

points. The free dynamics corresponding to the first normal mode, i.e. bending, of each resulting model has been simulated. The windowing approach has been applied on the time series data by using a length w_l of 100 time samples and $o_w = 1$. The reconstructed time history of the tip displacement of a model within the range of samples is shown in Figure 9.

Validation: As first attempt to the problem, the interface degradation has been simulated by interpolating linear solutions over time. The reference tip force vs displacement curve is shown in Figure 10 and compared with the reconstructed curve obtained through the trained VAE model.

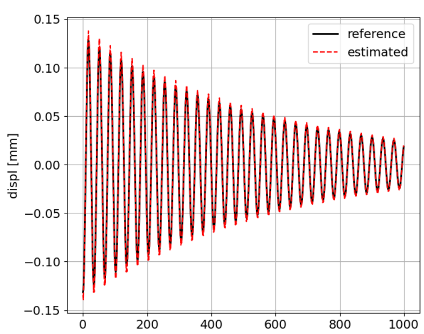

Figure 9. Tip displacement of DCB. Sample in the training set.

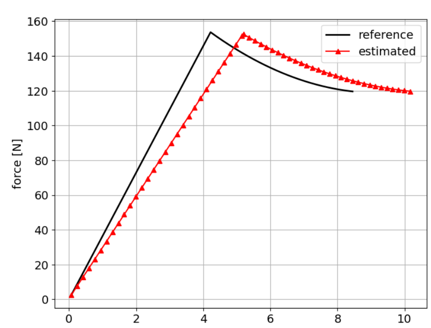

Figure 10. Reconstruction of force vs displacement curve.

CONCLUDING REMARKS

The identification of non-linear forces and parameters has been addressed in this article by employing data-driven and physics-informed ML approaches. In particular, LSTM and VAE are compared on a non-linear cantilever beam. Both models provide a good accuracy on the full-field displacement reconstruction. The chosen architecture of the LSTM model can not allow the identification of non-linear forces. A trainable decoder has been tested within the VAE model but a lack of physical constraints did not allow a good reconstruction and the standard POD approach shows to be the best case scenario for MOR. A novel approach of VAE for time-dependent parameter degradation has been also investigated for a DCB model. The methodology allows to reconstruct non-linear behaviors by interpolating the information given by the dataset of FE linear models. The investigated case of study is a simplified version of a real DCB model but the methodology can be generalized for more sophisticated mechanical system.

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