Incorporating Modal Testing into Dynamic Load Identification from Structural Vibration Measurement

YUANCHEN ZENG, ALFREDO NÚÑEZ and ZILI LI

ABSTRACT

Load identification from vibration measurements is usually cost-effective for obtaining dynamic loads on structures. Modal testing enables structural models to be validated or updated prior to being used for load identification. This paper proposes a methodology to make better use of modal test data for developing load identification methods. Modal test data are split into two datasets. One is used for model updating, and the other is used to validate load identification methods by inputting the measured vibration response and comparing the identified loads with the measured loads. Cross-validation can be achieved by using different splitting of data. This paper demonstrates this methodology with a case study in which impact hammer tests are performed on a stiffly-connected assembly. The test data support the optimization-based updating of a simplified model and the validation of a proposed Newmark-β-based load identification method. The results show that different impact locations of the model updating dataset affect the identification accuracy of the load identification dataset. The Newmark-β-based method yields comparable performance to the traditional frequency-domain method, while it has the advantage of being implemented at each time step.

INTRODUCTION

Obtaining dynamic loads on a structure is essential for assessing its health condition and predicting its remaining useful life. Since forces are often more difficult or expensive to measure than responses, identifying dynamic loads from vibration measurements is usually cost-effective, and many methods have been developed. In general, these methods combine analyses of measured structural responses with characteristics of structural dynamics in the frequency or time domain [1]. Frequency-domain methods are based on spectra analysis, in which structural dynamics is usually characterized by transfer functions or frequency response functions (FRFs) [2~4]. Time-domain methods enable loads to be identified at each time step, which is more applicable to non-stationary loads. In different time-domain methods, structural dynamics is characterized by different types of models, such as mass/stiffness/damping matrices [5, 6], modal parameters [5, 7], impulse response functions [8~10], state transition matrices [11], and surrogate models [12, 13].

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Modal testing is widely applied to validate or update structural models before they are used for load identification [9, 10, 14]. It enables more accurate characterization of structural dynamics and potentially more accurate load identification. In the literature, modal test data has rarely been used to validate load identification methods.

This paper aims to incorporate and make better use of modal test data for developing and validating load identification methods. First, a general methodology will be introduced, which splits modal test data into a model updating dataset and a load identification dataset. Then, an experimental case study will be presented, in which impact hammer tests on a stiffly-connected assembly will support the updating of a simplified model and the validation of a proposed Newmark-β-based load identification method. Finally, conclusions will be drawn.

METHODOLOGY

Figure 1 illustrates how modal testing is incorporated into the development and validation of load identification methods. In modal tests, a structure is excited by artificial loads at different locations, and its vibration responses are measured at different locations. All modal test data (including loads and responses) are then split into a model updating dataset and a load identification dataset. The model updating dataset enables the structural dynamics to be characterized and further a model of the structure to be updated. Then, by combining the updated model with a load identification method, loads on the structure can be identified from arbitrary vibration measurements. The method can be tested by inputting the measured vibration response in the load identification dataset and comparing the identified loads with the measured loads. Such a procedure can be repeated with different splitting of the modal test data to cross-validate the load identification method. Finally, the model of the structure can be updated using the complete modal test data, and further tests of the load identification method can be performed.

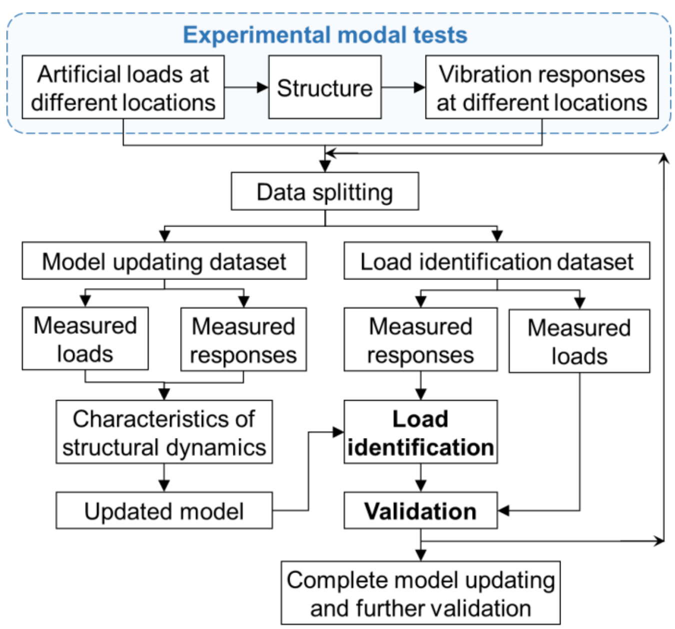

Figure 1. Methodology for incorporating modal testing into load identification.

In the proposed methodology, modal testing provides not only an updated model for driving load identification but also experimental validation of load identification methods. In particular, some artificial loads in modal testing, such as impact loads, are significantly transient, and they allow load identification methods to be assessed in challenging situations with such transient loads.

EXPERIMENTAL CASE STUDY

Modal Tests on A Stiffly-Connected Assembly

We showcase the above methodology using a mechanical assembly, as shown in Figure 2. It is intended to resemble a vehicle on the V-Track test rig [15]. The assembly consists of an upper mass connected to a beam through preloaded bolts and a lower mass suspended below the upper mass through springs. In some situations, an additional jack-loading frame is mounted to connect the two masses and preload the springs. In this case, the two masses and the beam are stiffly connected with high damping due to friction from the high preloads. Figure 3 (a) illustrates the dynamics of the assembly. Additionally, the presence of nonlinearities and local resonances further increases the challenge of the load identification.

We conduct impact hammer tests on the assembly using a Brüel & Kjær 8206-003 hammer. As shown in Figure 2, impact loads are generated and measured at two locations – one on the upper mass and one on the lower mass. Meanwhile, we measure the vertical vibration of the assembly using six PCB 356B21 accelerometers – three on the upper mass and three on the lower mass. The sampling rate is 102,400 Hz. Three groups of tests are conducted, as listed in Table I. Following the proposed methodology, two groups of data are used to update a model of the assembly separately, and the remaining is used to validate a load identification method based on the updated models. The data splitting in Table I is used as an example, while other different strategies are also applicable. The FRFs of each sensor location (u_i and l_i, i=1, 2, 3) for each impact load (p_1 and p_2) are averaged between the two repeated tests of each group, denoted as $G_{p_1u_i}(f)$, $G_{p_1l_i}(f)$, $G_{p_2u_i}(f)$, and $G_{p_2l_i}(f)$, and f denotes the frequency.

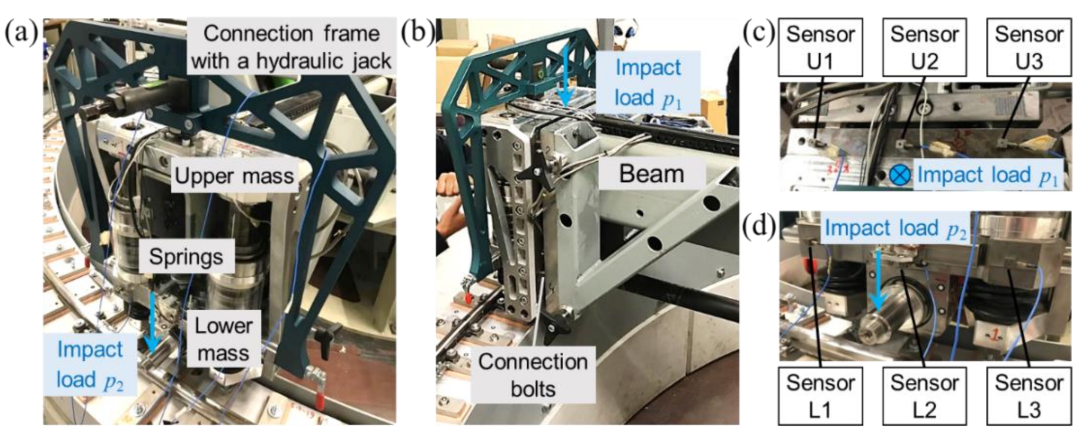

Figure 2. A stiffly-connected assembly and impact and sensor locations in modal testing. (a) Front view; (b) Rear view; (c) The upper mass; (d) The lower mass.

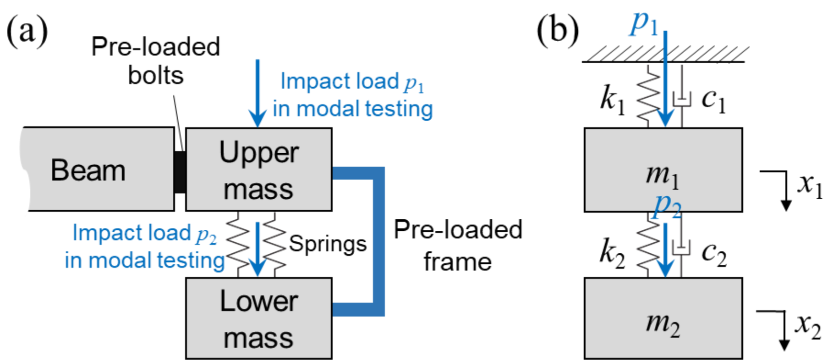

Figure 3. (a) Illustration of the assembly dynamics; (b) Simplified model of the assembly.

TABLE I. SPLITTING OF MODAL TEST DATA

Group number	Impact location	Number of repeated tests	Use of data
1	Upper mass	2	Update a model based on modal tests with impacts only on the upper mass
2	Lower mass	2	Update a model based on modal tests with impacts only on the lower mass
3	Upper mass	2	Validate the identification of impact loads on the upper mass

Optimization-based Model Updating

Load identification is usually driven by a model representing the dynamics of the structure. The more accurate the model, the better the representation of the structural dynamics. However, as model complexity increases, model updating and load identification become more challenging because ill-posedness and local minima become more problematic in solving such inverse problems. Therefore, we use a two-degree-of-freedom (DOF) model to represent the dynamics of the assembly, as shown in Figure 3 (b). All parameters of the model (m_1, m_2, k_1, k_2, c_1, c_2) are assumed to be unknown. These parameters are not equal to the physical parameters of the assembly but represent the equivalent parameters in the simplified and linearized model.

The transfer functions from each of the two impact loads (p_1, p_2) to the displacements of the two masses (x_1, x_2) can be calculated by applying the Laplace transform to their equations of motion, denoted as $H_{p_1x_1}(f)$, $H_{p_1x_2}(f)$, $H_{p_2x_1}(f)$, and $H_{p_2x_2}(f)$. Then, we identify the parameters of the model by minimizing the sum of squared logarithmic deviations between the measured FRFs (after being averaged) and the calculated transfer functions, as expressed below.

$$\left(m_1^*, m_2^*, k_1^*, k_2^*, c_1^*, c_2^*\right) = \arg\min \sum_{\substack{f\in \mathrm{F} \\ j\in \mathrm{J}}} \left(\log^2 \frac{3\left|H_{p_j x_1}(f)\right|}{\sum_{i=1}^{3}\left|G_{p_j u_i}(f)\right|} + \log^2 \frac{3\left|H_{p_j x_2}(f)\right|}{\sum_{i=1}^{3}\left|G_{p_j l_i}(f)\right|} \right) \tag{1}$$

where F represents the frequency ranges of FRFs for model updating and J represents the impact locations of modal tests used for model updating.

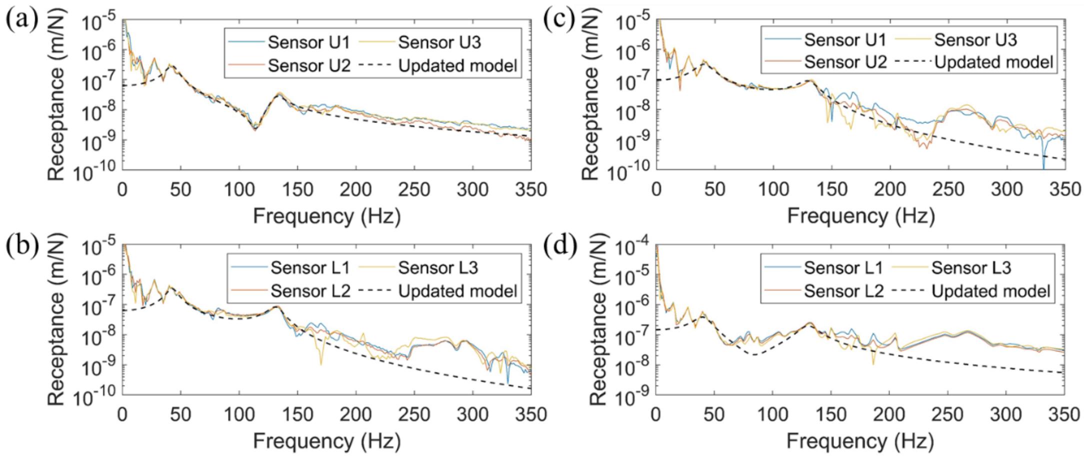

Figure 4. Modal testing and model updating results using the first two groups of data. (a) From p_1 to x_1; (b) From p_1 to x_2; (c) From p_2 to x_1; (b) From p_2 to x_2.

We employ the constrained optimization solver of sequential quadratic programming [16] in MATLAB to find the optimal model parameters within a given range for each parameter. These ranges are very wide because no prior knowledge of these parameters is available. Initial parameters are randomly generated within the given ranges. We repeat such random initialization and iterative optimization to find the (quasi-) global optimum from multiple local optima.

First, the measured FRFs from the first group of modal test data are used to update the model parameters, i.e., J={1} in Equation (1). We use frequencies near the resonance peaks for model updating, i.e., F={35 Hz<f<75 Hz}∪{120 Hz<f<145 Hz}. The measured FRFs and the corresponding transfer functions calculated from the updated model are plotted in Figure 4 (a) and (b). Similarly, the modal testing and model updating results based on the second group of data (J={2}) are shown in Figure 4 (c) and (d). The results show that the updated models provide transfer functions that are very close to the measured FRFs, especially near the resonance peaks at 40 Hz and 130 Hz. The two resonance peaks correspond to the in-phase and anti-phase bouncing of the two masses. This consistency demonstrates the effectiveness of model updating.

Some deviations can be observed outside the frequency ranges for the model updating. In the measured FRFs, the peaks below 35 Hz come from the bending of the beam, and those above 200 Hz may be related to local resonances of the lower mass. Since these dynamics are not included in the model, the transfer functions from the updated models cannot well represent them.

Newmark-β-based Load Identification

Newmark-β algorithm is an implicit and robust method for numerical integration [17] and has been adapted into a few load identification methods [18, 19]. In [18], a vector containing the loads at all time steps is computed for each identification, so the dimension of equations increases with increasing time step. In [19], loads can be identified iteratively at each time step with part of the responses unknown, but the need for displacement responses reduces its applicability in many situations.

In this paper, we develop a Newmark-β-based method to achieve step-wise load identification from complete acceleration measurements under constrained loading conditions. Generally, structure dynamics are characterized by mass, stiffness, and damping matrices, denoted as **M**, **K**, and **C**. We denote the displacement vector and load vector as $\boldsymbol{x}$ and $\boldsymbol{p}$, respectively, and the j-th (j=1, …, n) element of $\boldsymbol{x}$ as x_j. Among the n DOF, some may be known not to be directly loaded by external forces, i.e., the corresponding elements in $\boldsymbol{p}$ are zero. We include the indices of these DOF in a set B and the rest in its complementary B^{C}. If no such information is available, B=∅, and the identified loads are non-zero for all DOF. Then, the loads on the structure at each time step $\boldsymbol{p}(s)$ (s=0, 1, …) can be identified from the accelerations of all DOF $\ddot{x}$ as follows.

Step 1: Set s=0 and define initial conditions $\boldsymbol{x}[0], \dot{\boldsymbol{x}}[0], \ddot{\boldsymbol{x}}[0]$.

Step 2: Define time step size Δt and integration parameters β and γ [17].

Step 3: Calculate the equivalent stiffness matrix as follows [17].

$$\mathbf{K}_{\mathrm{e}} = \frac{1}{\beta \Delta t^2}\mathbf{M} + \frac{\gamma}{\beta \Delta t}\mathbf{C} + \mathbf{K} \tag{2}$$

Step 4: Estimate part of the displacement vector (corresponding to DOF with non-zero loads, i.e., x_j, $j \in \mathrm{B}^{\mathrm{C}}$) for the next time step by solving the following equations.

$$x_j[s+1] = \beta \Delta t^2 \left(\ddot{x}_j[s+1] + \frac{1}{\beta \Delta t}\dot{x}_j[s] + \frac{1-2\beta}{2\beta}\ddot{x}_j[s] \right) + x_j[s] \quad j = 1,\ \ldots,\ n \tag{3}$$

$$\begin{aligned} \sum_{h=1}^{n} K_{\mathrm{e}(j,h)} x_h[s+1] &= \sum_{h=1}^{n} M_{(j,h)} \left(\frac{1}{\beta \Delta t^2} x_h[s] + \frac{1}{\beta \Delta t}\dot{x}_h[s] + \frac{1-2\beta}{2\beta}\ddot{x}_h[s] \right) \\ &\quad + \sum_{h=1}^{n} C_{(j,h)} \left(\frac{1}{\beta \Delta t} x_h[s] + \frac{\gamma-\beta}{\beta}\dot{x}_h[s] + \frac{\Delta t(\gamma - 2\beta)}{2\beta}\ddot{x}_h[s] \right) \end{aligned} \quad j \in \mathrm{B} \tag{4}$$

where $K_{\mathrm{e}(j,h)}$ represents the j-th row and h-th column of $\mathbf{K}_{\mathrm{e}}$, and the same notation applies to **M** and **C**. If B=∅, Equation (4) vanishes, and the entire displacement vector $\boldsymbol{x}[s+1]$ can be calculated according to Equation (3). If B≠∅, overdetermined equations of $x_j[s+1]$, $j \in \mathrm{B}^{\mathrm{C}}$ can be constructed by substituting $x_j[s+1]$, $j \in \mathrm{B}$ obtained from Equation (3) into Equation (4). These overdetermined equations can be solved using least square methods, in which regularization can be included [20]. Further, the remaining displacement $x_j[s+1]$, $j \in \mathrm{B}$, can be obtained by solving Equation (4) after substituting $x_j[s+1]$, $j \in \mathrm{B}^{\mathrm{C}}$. By doing so, the constraints of loading conditions are satisfied.

Step 5: Estimate the velocity vector $\dot{\boldsymbol{x}}[s+1]$ and the load vector $\boldsymbol{p}[s+1]$ as follows.

$$\dot{\boldsymbol{x}}[s+1] = \dot{\boldsymbol{x}}[s] + \Delta t(1-\gamma)\ddot{\boldsymbol{x}}[s] + \gamma \Delta t \ddot{\boldsymbol{x}}[s+1] \tag{5}$$

$$\begin{aligned} \boldsymbol{p}[s+1] &= \mathbf{K}_{\mathrm{e}}\boldsymbol{x}[s+1] - \mathbf{M}\left(\frac{1}{\beta \Delta t^2}\boldsymbol{x}[s] + \frac{1}{\beta \Delta t}\dot{\boldsymbol{x}}[s] + \frac{1-2\beta}{2\beta}\ddot{\boldsymbol{x}}[s] \right) \\ &\quad - \mathbf{C}\left(\frac{1}{\beta \Delta t}\boldsymbol{x}[s] + \frac{\gamma-\beta}{\beta}\dot{\boldsymbol{x}}[s] + \frac{\Delta t(\gamma-2\beta)}{2\beta}\ddot{\boldsymbol{x}}[s] \right) \end{aligned} \tag{6}$$

Step 6: Increase the time step s by 1 and repeat Steps 4~5 for the new time step.

Validation of Load Identification

Based on the updated two-DOF model, the mass, stiffness, and damping matrices can be determined, and the displacement and load vectors are $\boldsymbol{x}=[x_1, x_2]^T$ and $\boldsymbol{p}=[p_1, p_2]^T$, respectively. The third group of modal test data is used to validate the Newmark-β-based load identification method. Since we know that the impact load is applied on the upper mass, the constraint of $p_2=0$ is included. The measured acceleration responses are averaged over U1 and U3 for the upper mass and over L1 and L3 for the lower mass. A low-pass infinite impulse response filter with a cut-off frequency of 1,000 Hz is applied to reduce noise in the raw signals.

Figure 5 presents the load identification results for the two tests in this dataset, consisting of a light impact and a heavy impact. In each plot, the load measured by the force transducer in the hammer is plotted as a reference. The identified loads using the model updated with impacts on the upper mass p_1 (the first group of data) and those using the model updated with impacts on the lower mass p_2 (the second group of data) are plotted. Moreover, we apply the traditional frequency-domain load identification method [1] to the same acceleration responses, and the results are also shown. No regularization is used in the two identification methods.

The results show that the impact loads can be effectively identified from the measured accelerations through the two methods when using the model updated with impacts on the upper mass. Both methods underestimate the impact loads when using the model updated with impacts on the lower mass, reflecting the influence of different impact locations in the modal tests on the load identification performance. Outside the impact phase, some oscillatory errors can be observed, mainly due to the local resonance of the lower mass. In general, the identification results of the two methods are close to each other. Compared to the frequency-domain method, the proposed Newmark-β-based method has the advantage of being implemented at each time step.

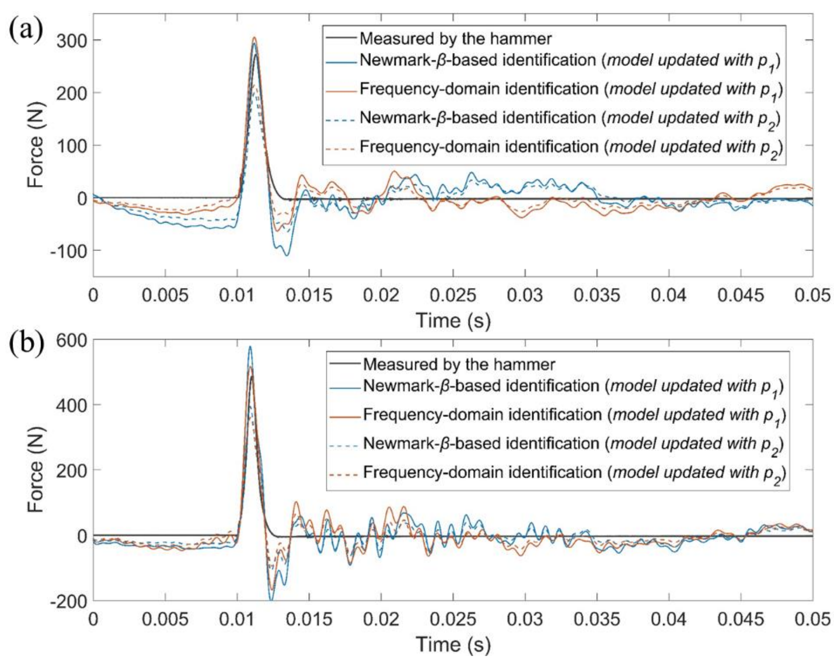

Figure 5. Comparisons between the measured loads and the identification results of different methods. (a) A light impact load; (b) A heavy impact load.

CONCLUSIONS

This paper proposes and demonstrates how modal test data are split and used to update structural models and validate load identification methods. This methodology provides a convenient experimental assessment of load identification methods and enables cross-validation with different data splitting. In future research, we will improve the applicability of the proposed Newmark-β-based method to incomplete acceleration measurements and further test it on structures with multiple loading positions.

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