Separation of Physical from Mathematical Poles in Operational Modal Analysis Using Reduced, Reassembled Covariance Matrices

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ABSTRACT

Modal analysis is one of the elementary tools for the analysis of structural dynamics. In context of vibration-based damage diagnosis and continuous monitoring of modal quantities, different variants of Operational Modal Analysis (OMA) were developed. In most cases, the results of the analysis (modal data) are improved by post-processing methods, e.g., by clustering of stable poles in stability diagrams. The aim of this paper is to facilitate the automatic interpretability of the computational results by pre-processing techniques. The method presented in this paper is used to distinguish between the physical poles and the unwanted mathematical poles to determine the natural frequencies more accurately. This is necessary because for the monitoring of complex real structures, high model orders are required, leading to the occurrence of mathematical poles. To enable a reliable separation, the spectral signal components are analyzed regarding their information content and their energy level in a specific frequency band. For this purpose, the widespread assumption is used that the largest singular values of the system response are related to the highest signal energy and that the lower singular values are caused by noise. With the use of different narrow frequency bands (e.g., by using bandpass filters), the estimation of an accurate threshold between noise and eigenfrequencies can be well established for each band. This technique is applied on the Covariance-Driven Stochastic Subspace Identification algorithm (SSI-COV) for OMA. In context of SSI-COV the Singular Value Decomposition (SVD) and the estimated threshold is used to build reduced and reassembled Hankel matrices from partial sum matrices for each frequency band separately. The advantage of the reduced matrices is that only physical poles have plausible modal damping values, and these can be well separate from mathematical poles. The effectiveness of the method is first demonstrated on simulated data and then successfully tested on a laboratory structure. The results, the advantages, and limitations of the method as well as the need for further improvements are discussed.

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INTRODUCTION

The OMA is a widely used method for system identification in the area of Structural Health Monitoring (SHM) [1]. It can identify eigenfrequencies of a structure, the corresponding damping values and mode shapes. There are different techniques which can be used for the estimation of the wanted parameters, e.g. SSI-Data, Peak-Picking or Polymax to name some. Each of them has its own advantages and disadvantages [2]. This paper only deals with the SSI-COV algorithm. The approach in this work was developed as part of an automated OMA to be used in SHM context [3,4]. The underlying idea is to facilitate the separation of mathematical poles from eigenfrequencies in such a way that the effort and the complexity of post-processing methods for the interpretation of OMA results can be reduced. Therefore, the well-known gap-method is modified and applied to estimate the required model order only for narrow frequency bands [5]. Due to the use of narrow frequency bands, which rarely contain more than one or two eigenfrequencies, the determination of the necessary order is simplified. All narrow frequency bands together cover the whole frequency spectrum. The defined model order is then used to build a reduced and reassembled Hankel matrix from partial sum matrices for each frequency band separately. The results calculated in this way together with the application of hard criteria (e.g., limitation for permissible attenuation values) only contain physical modes. The procedure is first tested as an example on simulated data and then, for reasons of space, only applied to a damage mode of a laboratory setup.

FUNDAMENTALS OF SSI-COV

The following chapter gives a brief overview of the SSI-COV algorithm so that the effects of the herein proposed method can be better comprehended. A detailed explanation can be found in the publication of Rainieri and Fabbrochino [6]. The algorithm belongs to the output-only methods for linear system identification, originally based on the eigenvalue realization algorithm designed for known impulse excitation. [7] In case of SSI-COV the structure response is "modeled" as a state-space model and the unknown excitation is assumed to be uniformly distributed white noise. In practice the amplitude or excitation distribution is unknown, which means that modes in various frequency bands can be stimulated differently. That can make a reliable system identification quite difficult especially when considering a wide frequency range with different excited modes. The core of SSI-COV is the construction of the output covariance matrix in block Hankel structure Eq. (1), which contains the results of the cross and auto covariance functions between the sensor signals Eq. (2). The variables $\tilde{\beta}, \tilde{\alpha}$ from Eq (1) correspond to the maximum number of time steps of the correlation function. The variable n_t corresponds to the number of data points in one signal of the simultaneously measured m-variate time series y of the dimension [m, nt]. This leads to a covariance matrix $\hat{R}i$ of dimension [m, m].

$$(H_{\alpha,\beta})_0 = \begin{bmatrix} C_y G & C_y A_d G & \cdots & C_y A_d^{\beta-1} G \\ C_y A_d G & C_y A_d^2 G & \dots & C_y A_d^{\beta} G \\ \vdots & \vdots & \ddots & \vdots \\ C_y A_d^{\alpha-1} G & C_y A_d^{\alpha} G & \dots & C_y A_d^{\alpha+\beta-2} G \end{bmatrix} = \begin{bmatrix} \hat{R}_1 & \hat{R}_2 & \cdots & \hat{R}_{\tilde{\beta}} \\ \hat{R}_2 & \hat{R}_3 & \cdots & \hat{R}_{1+\tilde{\beta}} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{R}_{\tilde{\alpha}} & \hat{R}_{1+\tilde{\alpha}} & \cdots & \hat{R}_{\tilde{\alpha}+\tilde{\beta}-1} \end{bmatrix} \quad (1)$$

$$\hat{R}_i = \frac{1}{n_t\text{-}i\text{-}1} \; y_{(1:nt-i+1)} \cdot y^T_{(1:nt)} \tag{2}$$

For further calculation steps it is necessary to decompose the $[m \cdot \tilde{\alpha}, m \cdot \tilde{\beta}]$ Hankel matrix (H_o) into two orthogonal matrices U and V and a diagonal matrix S by means of SVD, see Eq. (3).

$$U_p = \begin{bmatrix} u_{11} & \cdots & u_{1p} \\ \vdots & \ddots & \vdots \\ u_{1m} & \cdots & u_{mp} \end{bmatrix} \; S_p = \begin{bmatrix} \sigma_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{np} \end{bmatrix} \; V_p = \begin{bmatrix} v_{11} & \cdots & v_{1p} \\ \vdots & \ddots & \vdots \\ v_{1m} & \cdots & v_{mp} \end{bmatrix} \tag{3}$$

The dimension of the Hankel matrix determines the maximum model order and has a significant influence on the calculation of the physical poles. Weakly excited modes can only be detected by a higher model order and consequently a high dimensional Hankel matrix. The drawback is then that the stability diagram will also contain more spurious or mathematical poles. The system matrix A_d can now be obtained by Eq. (4)

$$A_d = S_p^{-1/2} \cdot U_p \cdot H_1 \cdot V_p \cdot S_p^{-1/2} \tag{4}$$

LOW RANK APPROXIMATION OF HANKEL MATRIX

A simulated system with n degrees of freedom has n eigenfrequencies. These eigenfrequencies are distributed over an arbitrarily large frequency range. The modal analysis can be used to identify these eigenfrequencies within the analyzed frequency range. Each eigenfrequency corresponds to two singular values in the singular value matrix S. Due to lack of space, the new approach is explained by means of an example. Figure 1 shows a simulated three degree of freedom system (3-DOF System). For this system we expect to see three natural frequencies and therefore six dominant singular values. These singular values should be clearly distinguishable from the remaining singular values, which are caused by a too high model order or measurement noise. In Figure 2, the singular value plot of the first 15 singular values is shown for the 3-DOF System with an unusually high number of timeshifts (300). It is easy to see that despite the possibility of 900 singular values, 99% of the total energy is covered by the first 6 singular values. The remaining singular values contain less than 1% of energy and can therefore be omitted.

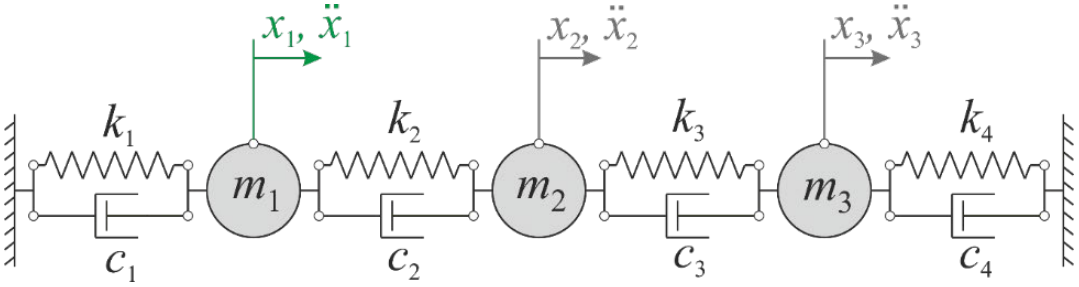

Figure 1. 3-DOF Mass-spring-damper model

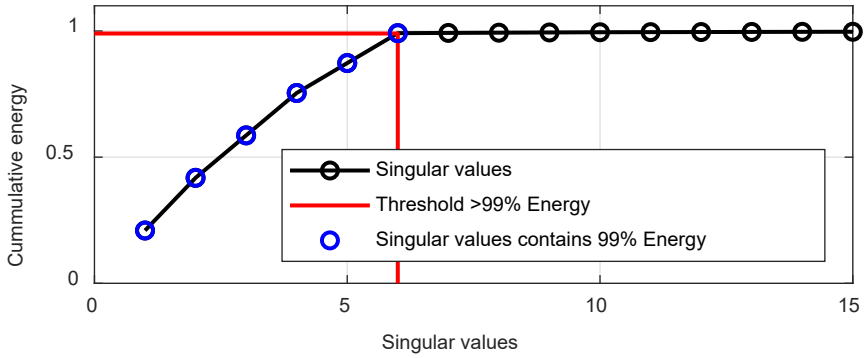

Figure 2. Singular value curve of 3DOF-System

According to the literature or the commonly known SSI-COV algorithm, a model order of $p = 6$ would now be selected to calculate the A_d matrix Eq. (4) p times. In this approach however, a new reference matrix $H_{1\mathrm{rec}}$ is first built from the partial sum of p rank one matrices, as shown in Eq. (5). All further singular values greater than p are ignored. The use of partial sums of rank one matrices is often applied in the context of signal denoising or image compression [8–11].

$$H_{1\mathrm{rec}} = \sum_{i=1}^{p} s_i u_i v_i^T = s_1 u_1 v_1^T + s_2 u_2 v_2^T + \cdots + s_k u_k v_k^T \tag{5}$$

The advantage of this method is that the high computational order, typically required for the A_d matrix to determine weakly excited modes, can be maintained, and mathematical poles no longer have a significant impact on the interpretation of the stability diagram or modal parameters. Through the low-rank approximation, only the significant information of the natural frequencies remains in the $H_{1\mathrm{rec}}$ matrix. The H_0 matrix is not changed, so it still contains all the information. The procedure for calculation of the system matrix A_d with Eq. (4) remains unchanged. Figure 3, left, shows the poles calculated using the regular method, while Figure 3, right, shows the poles obtained using the low-rank approximation with six singular values. In the regular method, most of the mathematical poles are located within a plausible damping range and may therefore appear as stable or physical poles (occurrence in multiple orders, low frequency-damping deviation). In this approach based on the low-rank approximation, mathematical poles still occur, but they are much more random in terms of their frequency distribution. Additionally, the modal damping of these mathematical poles is close to 100%. These poles can be easily removed by means of a simple damping criterion, e.g. removing poles with damping greater than 10%.

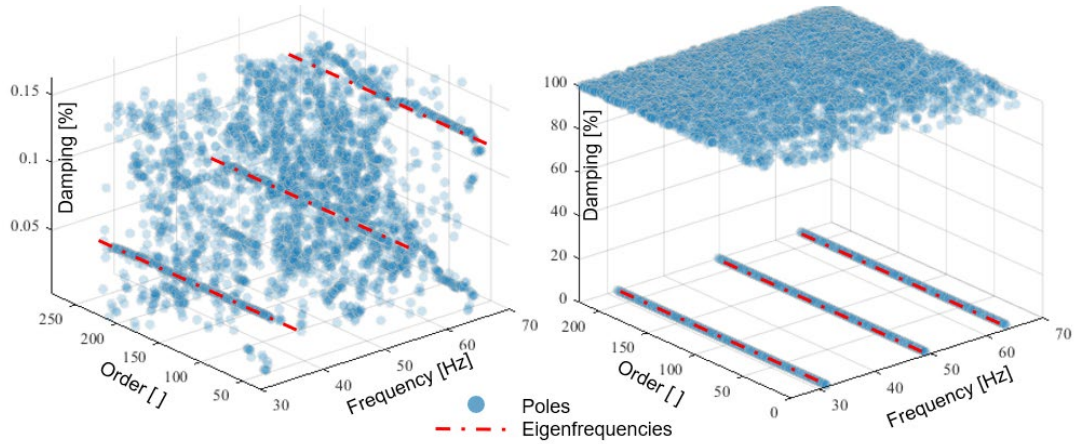

Figure 3. Regular calculation (left), modified approach without damping criteria (right)

APPLICATION TO MEASUREMENT DATA OF A LABORATORY DEMONSTRATOR

One weakness of the presented method is the determination of the correct model order based on the singular value decay. While this is easy for simulated data with low noise, it is not easily achievable for a real system with many eigenfrequencies and disturbances. In the following, the method is applied to measurement data of a laboratory structure that was created as part of research project In-Situ WIND. The structure serves as a replica of the so-called grout connection in offshore structures, such as those used for connecting foundation structures and superstructures in windturbines. To test different SHM approaches in presence of different induced damage stages, the structure is gradually loaded with hydraulic cylinders. The structural response is recorded using four acceleration sensors, which are mounted in the load direction. The measurements are taken between load levels using an electromechanical shaker to excite the demonstrator with different forms of excitation (noise, sweep, etc.). The frequency range of the excitation is relatively high (up to 1000 Hz), which can excite numerous modes. Since a clear model order cannot or only poorly be determined based on the singular value decay, modal analysis is selectively performed in this case. This means that the frequency range of interest from 0 to 1000 Hz is divided into many narrow frequency bands using a bandpass filter, for which an individual low order p for the low rank approximation is determined, see Figure (5). Due to lack of space only one specific frequency band is used, to show the effectiveness of the proposed method. To obtain the results for other modes, different bandpass frequencies or sliding windows over the frequency band, must be used. In the following, for illustration we only consider one mode which allowed us to detect a degradation of the test stand in an early stage and after running various load programs. For this purpose, the acceleration signals are first filtered using a bandpass filter before these are used for the estimation of modal parameters. The passband frequency here is 225 Hz up to 270 Hz (vertical red dashed lines) in Figure 4.

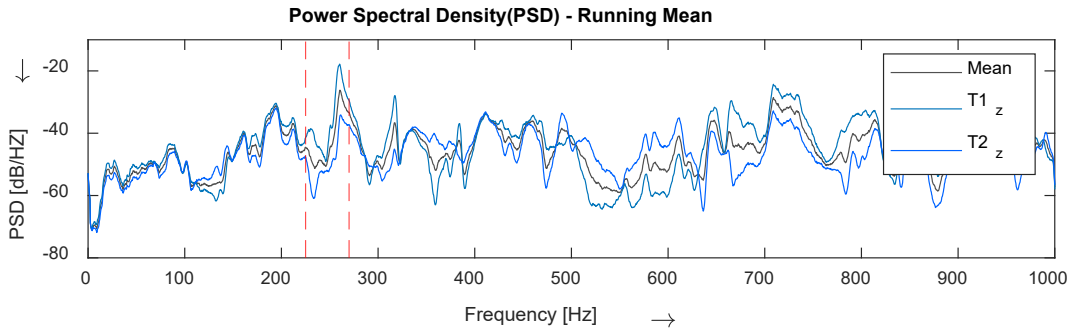

Figure 4. Power Spectral Density over the whole frequency band. (Vertical red dashed lines –bandpass frequency)

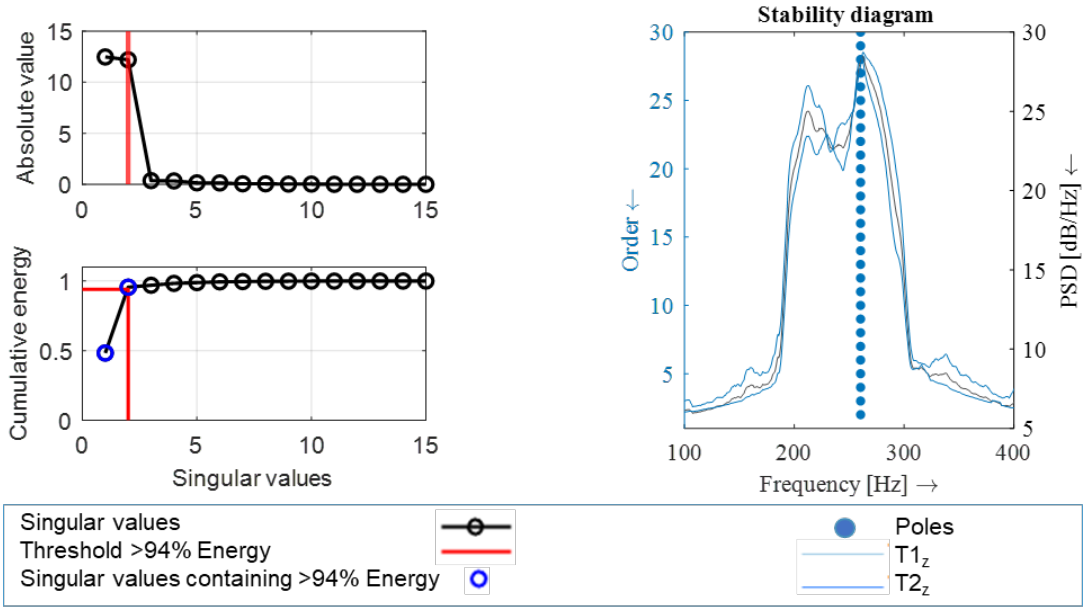

Figure 5. Singular values (left) and stability diagram with marked stable poles (right)

According to Figure 5 (left), the analysis of the singular value plot shows two relevant singular values of the filtered frequency band shown in Figure 6 (right). Therefore, there is only one corresponding mode in this frequency range, which can be represented by the low rank approximation with p = 2 according to Eq. (5) for the H_{1rec} matrix. By using the newly constructed Hankel matrix H_{1rec} and the mentioned hard damping criterion, all remaining mathematical poles are removed, leaving only the relevant poles of the natural frequency in the chosen band. The remaining poles are marked as blue dots in Figure 5 (right). With these determined parameters, all further data sets, which were recorded after each hydraulic load level, during the shaker excitation, are calculated and compared with the previous one.

TABLE I EVALUATION OF THE MODAL ANALYSIS OF DAMAGE MODE

Date	Load level	Frequency in [Hz]	Difference to previous experiment [Hz]	Damping in [%]	Difference to previous experiment [%]
31.10.2022	Reference	260,41	-	0,5	-
03.11.2022	Initial commissioning 10 %	258,46	-1,95	0,52	0,02
16.11.2022	5 Hours 10 %	254,43	-4,03	1,02	0,5
18.11.2022	5 Hours 20 %	251,72	-2,71	1,37	0,35
22.11.2022	5 Hours 40 %	247,59	-4,13	2,93	1,56

According to the results of the modal analysis as given in Table I, a clear reduction in the natural frequency and an increase in damping can be observed, indicating a loss of stiffness and damage to the structure. Based on these findings, it was determined that the degradation of the grout connection had already occurred after the first load level. No plastic deformation was observed in the steel components, suggesting that the damage was confined to the grout material itself. The suspicion was confirmed as the load level was increased, with the appearance of cracks in the grout joint and water from the inner pipe being forced upward due to a yet undetermined effect. The residual load-bearing capacity is still sufficient, a lowering of the inner pipe could not be observed.

CONCLUSION - LIMITATIOS AND ADVANTAGES OF THE CHOSEN APPROACH AND PERSPECTIVES FOR FURTHER RESEARCH

The presented approach refers to the use of a low rank approximation of the Hankel matrix $H_{1\text{rec}}$ to separate mathematical poles from physical poles when dealing with modal analysis. This method has been successfully tested both on simulated and experimental data. However, one of the main limitations of this approach is that it can be challenging to determine the unique model order for the entire frequency range. To overcome this issue, the whole frequency range is divided into several narrowband regions, and the model order is determined based on the singular value decay in each range. The novelty of this approach is that it allows a clear identification of the eigenfrequencies in each frequency band, without additional post-processing methods such as clustering, also the determination of calculation parameters is simplified. However, this approach requires multiple calculations of modal parameters, leading to increased computation time. The modified modal analysis algorithm is particularly suitable for tracking a specific vibration mode explicitly. This can be useful in various fields where it is essential to detect and track changes in structural behavior over time. Additionally, this approach could be applied to other areas such as automotive,

aerospace, and civil engineering, where modal analysis is a crucial tool for understanding the dynamic behavior of structures and systems.

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