Physics-Informed Gaussian Processes for Wave Loading Prediction

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ABSTRACT

The quantification of wave loading is an important step within the estimation of fatigue accrual within offshore structures. The direct measurement of wave loads can be both challenging and expensive, often requiring the installation of bespoke systems if at all possible. The estimation of wave loads based on data from other sensors, more commonly found on offshore structures (e.g. wave radars) is therefore highly desirable. This paper presents an experimental study of a monopile structure within a wave tank, instrumented with accelerometers, strain gauges, a force collar, wave gauges and a velocimeter subject to a range of wave conditions. Here the dataset is used to construct models which combine data-based Gaussian process NARX models with linear wave theory. The novel model structures presented rely on only wave gauge data as an input and achieve improved predictive performance over purely data-based approaches across a range of wave states.

INTRODUCTION

The cyclic loading of an offshore structure due the motion of waves is a driving factor of fatigue accrual and therefore has a significant impact of the useful remaining life of the structure. The quantification of wave loading, through either measurement or prediction, presents its own set of challenges. The direct measurement of wave loads acting on offshore structures is rare, and where attempted it often requires the development and installation of bespoke systems [1]. Even when measurements may be available, these are generally at point locations and do not provide access to a distributed load over the structure. The prediction of wave loads across a structure, using data readily available from other sensors has the potential to provide access to wave loads where they cannot be measured and reduce the cost of implementing additional measurement equipment. This paper will focus on the utilisation of incoming wave height data, commonly available from wave radars across many offshore structures [2, 3].

The modelling of waves and prediction of wave loads acting on structures is chal-

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lenging and forms an extensive field of research [4,5]. The harsh offshore environments, highly variable conditions and complexity of phenomena such as vortex shedding and breaking waves makes the validation of physics-based models difficult. Attempts to represent the underlying physics have lead to the development of Computational Fluid Dynamics (CFD) models which have been shown to be effective in a range of wave loading prediction tasks [6–8]. A key challenge facing physics-based approaches, including CFD, is that as the complexity of the phenomena being captured grows, so too does the required model fidelity. For a case such as breaking wave dynamics [9], the resulting CFD model is computationally expensive and requires extensive resources (time, money and technical expertise) to validate.

An ability to model processes without complete understanding of the underlying physics has been a key driver of the development of data-based models. Here the relationship between variables may be learned directly, without prior knowledge of how a process may behave. Within the field of wave loading quantification, neural networks [10], Gaussian process NARX models [11] and Bayesian regression [12] have shown to be helpful tools for capturing the non-trivial relationship between flow conditions and wave force. Although effective when operating within the realm of previously observed conditions, a tendency to extrapolate poorly, often exhibiting unexpected behaviours, and a lack of insight in to how a model operates has posed an obstacle to their adoption within an industrial setting.

The field of physics-informed machine learning focusses on integrating physical knowledge with data-based methods in order to benefit from the advantages of either method used independently. The aim is to create a flexible model, able to provide a degree of insight in to its operation and extrapolate effectively within the limits of known physics. The means by which combined model structures are created presents an interesting and emerging research topic, overviews of which can be found in [13,14]. Within the wave loading community, Pena [15] used a CFD model in order to generate training data for a Generative Adversarial Network (GAN). The resulting model achieved comparable performance to the CFD simulation with significant reductions in runtime. Previous work of the authors [16] combined Morison's Equation with Gaussian process NARX models to increase predictive performance in cases of limited training data.

This paper has two main objectives: to model the wave load acting on a monopile using only data of incoming wave height, and to showcase how physical knowledge may be integrated within Gaussian process NARX models to improve performance and provide interpretability. Here, linear wave theory is used to improve predictive performance by providing an approximation of flow conditions as a model input. A key conclusion drawn is that although the physical knowledge included is imperfect, relying on a number of simplifying assumptions, it is still able to assist the combined final models.

In the next section, an overview of the experimental setup is presented, detailing the dataset and generation of wave states. The proposed physics-informed model structures are then defined, with specific focus on how linear wave theory is utilised for the estimation of flow conditions. Finally, the performance of the final models across a range of wave states is compared and discussed.

EXPERIMENTAL SETUP

A series of experiments were performed on a monopile structure within the wave tank at the Laboratory for Verification and Validation (LVV) with the aim of studying wave loading on offshore structures. Although within a controlled environment, in which the harshness and high variability of a true offshore environment is not fully captured, the dataset serves as a useful tool to develop and test model structures before progressing to real world implementation.

The wave tank had an operational volume $10.74m$ in length, $0.5m$ in width and $1m$ in depth. Although the tank had paddles capable of generating waves in either direction, one set of paddles was kept slack as to absorb incoming waves and reduce reflection of waves back along the tank as much as possible. A series of two wave gauges were used to capture the free surface of passing waves, whilst an Acoustic Doppler Velocimeter (ADV) was used to measure water particle velocity and acceleration. A diagram of the relative dimensions between measurement equipment is shown in Figure 1.

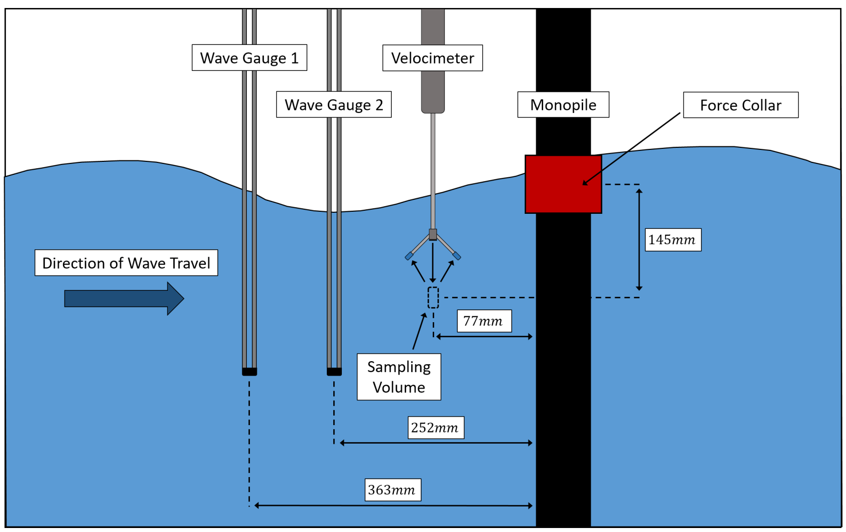

Figure 1: Relative positions of measurement equipment within the wave tank.

The structure was comprised of a $2990mm$ length of PVC pipe with an outer diameter of $90mm$ and wall thickness of $5.4mm$. In order to reduce the natural frequency of the structure and to mimic the dynamic behaviour of offshore monopile structures, mass was added to the top of the structure. The natural frequency was reduced from $2.46Hz$ to $0.8Hz$ via the addition of $8.45kg$ of mass.

The structure was instrumented with accelerometers, strain gauges and a force collar. Along with the measurement equipment within the wave tank, this would provide access to incoming wave heights, flow conditions, wave load, structure response and strain. The final dataset contained 27 channels, measured at 2048Hz.

Representation of Sea States

The generation of waves utilised representative sea spectra from the JOint North Sea WAve Project (JONSWAP) [17]. The JONSWAP spectrum is expressed:

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left(-\beta\left(\frac{\omega_p}{\omega}\right)^4\right)\gamma^a \tag{1}$$

where $\alpha = 0.076\left(\frac{U_{10}^2}{Fg}\right)^{0.22}$, U_{10} is the wind speed $10m$ above the surface, F is the fetch (the distance over which wind velocity remains constant), g is acceleration due to gravity, ω is angular frequency, ω_p is peak frequency, $\beta = 0.74$ and γ is the peak enhancement factor with exponent a.

By varying the parameters within the spectra, it is possible to define and generate a variety of ocean states. For this experiment, a series of JONSWAP waves were generated by varying both γ and ω_p, thereby creating a matrix of test conditions. Based on the capabilities of the wave tank, ω_p was varied from $0.7Hz$ to $1.1Hz$ in $0.1Hz$ increments and γ was varied from 1.3 to 5.3 in 1.0 increments. This was deemed to give a suitably fine grid of 25 wave states.

INTEGRATING PHYSICS AND DATA

The objective of constructed models is to predict wave load using incoming wave height as an input, however the relationship between free surface elevation and wave force is complex. This means that not only is a flexible model required, but also that this relationship is a highly challenging learning task, placing increased demand on required training data. The role of linear wave theory is to achieve an approximation of the flow conditions close to the monopile, thereby reducing the complexity of the learning task and aiming to increase performance at lower quantities of data. The change in model structure from a black-box approach is summarised in Figure 2.

The models developed will be used to predict the wave load acting on the monopile at the force collar, around the region close to the free surface. This is where flow velocities and accelerations are typically highest and where the maximum forces are likely to be experienced. The prediction of the largest forces applied to the monopile are the most important for the prediction of remaining fatigue life.

In this work, an autoregressive form of Gaussian Process Regression (GPR) is utilised as the data based component of the model, namely, a GP-NARX. An autoregressive model is a function of previous (lagged) versions of a target variable. They are useful when the behaviour of a variable is in some way dependant on the past behaviour of itself or other variables. There are many circumstances in which the inclusion of lagged variables are useful to aid the modelling of wave loads, including: capturing periodic trends e.g. dominant frequencies and periodicity of waves; capturing a delay e.g. upstream measurements; and representation on higher order terms e.g. approximation of phenomena such as vortex shedding [11, 18].

A Nonlinear AutoRegressive model with eXogenous inputs (NARX) is one which considers lagged additional inputs u_{t-i} as well as the lagged target y_{t-i}. These are then passed through a nonlinear function $f(x)$:

$$y_t = f([u_t,\ u_{t-1}, ...,\ u_{t-L_u},\ y_{t-1},\ y_{t-2}, ...,\ y_{t-L_y}]) + \varepsilon \tag{2}$$

where, within the context of wave loading prediction, the previous signal values, y_{t-i} are the wave force and the exogenous inputs u_{t-i} are typically data from other sensors.

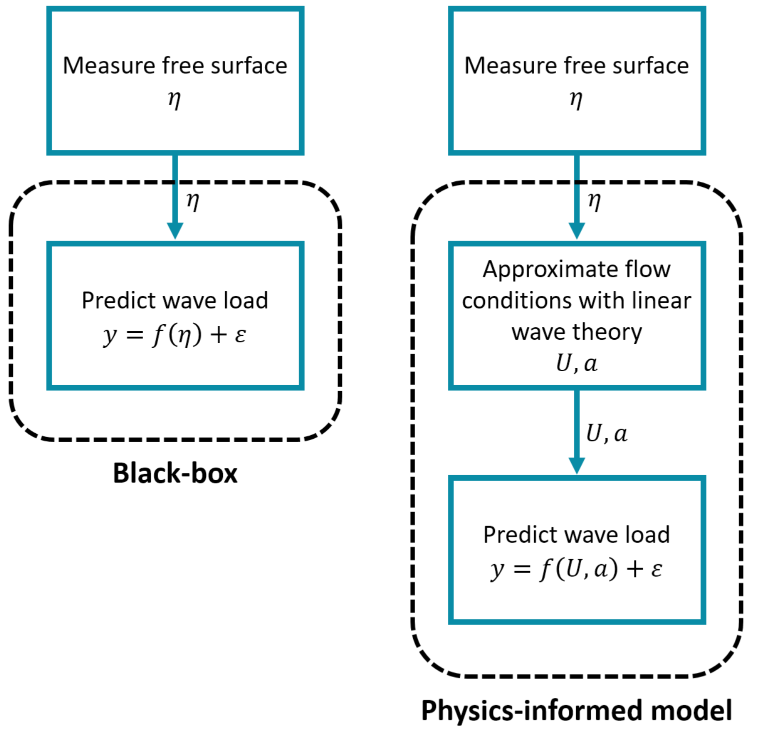

Figure 2: A comparison of the proposed physics-informed model structure with a black-box approach. The measured free surface η is used to approximate flow conditions (water particle velocity U and acceleration a) close to the monopile for use as an input to the model, rather than being directly used itself.

Here, for the purely data-based models, u_{t-i} will be measured wave gauge data, whilst for the physics informed models, u_{t-i} will be the approximated flow conditions. The determination of maximum lags L_u and L_y will affect model structure and performance; they should be selected via an appropriate lag selection process [16, 19, 20]. For this work, optimum lags of $L_u = 4$ and $L_y = 0$ were obtained following [16].

Within a GP-NARX, the nonlinear function $f(x)$ is a Gaussian Process (GP), offering several advantages over NARX models with a fixed functional form. Gaussian Process Regression (GPR) is a flexible, Bayesian, non-parametric machine learning tool that is effective within a wide range of SHM subtasks [21–23]. An overview of background GPR theory is provided within [24] for the interested reader. The primary motivators for its implementation here are a capability to model continuous functions without restriction to a fixed set of basis functions, helpful for capture of the complex relationship between fluid motion and wave load for which a closed functional form is not known; and a return of a full posterior distribution due to use of a Bayesian framework. An associated uncertainty with estimated loads has a significant impact on estimated on the estimated remaining fatigue life and its representation is therefore important [21, 25].

To integrate wave theory within the GP-NARX, the following workflow is proposed:

1. Perform a free surface reconstruction using the measured surface elevation from wave gauge 1, located $363mm$ from the monopile.

2. Construct a velocity field for the sum of linear waves obtained from the free surface reconstruction.

3. Use the velocity field to achieve an estimate for the flow conditions (water particle velocity U and acceleration a) at the force collar.

4. Use the flow conditions at the force collar as inputs within models for wave loading prediction, testing performance using the measured force data.

The reconstruction of the free surface follows the methods described within [26] and relies on the decomposition of the wave in to a sum of harmonic components. This allows for the expression of free surface η as a sum of N linear waves.

$$\eta = \sum_{i=1}^{N} A_i cos(k_i x - \omega_i t + \Phi_i) \tag{3}$$

where for the ith wave, A_i is amplitude, k_i is the wave number, ω_i is angular frequency and Φ_i is the phase. The measured free surface elevation from wave gauge 1 was used for the reconstruction, with an average NMSE of 15.48% achieved on the free surface fit at wave gauge 2 across all JONSWAP waves. None of the measured data from wave gauge 2 was shown to the model before performance was measured.

The reconstruction of the free surface required the decomposition of the wave in to a sum of linear waves, which provides a useful tool for the derivation of several properties [5]. An important one of which is the velocity potential ϕ, which when treated as a sum over N linear waves is expressed:

$$\phi = \sum_{i=1}^{N} \frac{A_i g}{k_i c_i} \frac{cosh(k_i(z+d))}{cosh(k_i d)} sin(k_i x - \omega_i t + \Phi_i) \tag{4}$$

where, additionally, g is acceleration due to gravity, d is water depth and c_i is wave speed. The water particle velocities are negative spatial derivatives of the velocity potential and their calculation for a given range of x and z will allow for the construction of a velocity field. Noting that velocities here are only meaningful below the free surface, the horizontal velocity U_x and vertical velocity U_z are expressed:

$$U_x = -\frac{\partial \phi}{\partial x} = \begin{cases} \sum_{i=1}^{N} \omega_i A_i \frac{cosh(k_i(z+d))}{sinh(k_i d)} cos(k_i x - \omega_i t + \Phi_i) & z \leq \eta \\ 0 & z > \eta \end{cases} \tag{5}$$

$$U_z = -\frac{\partial \phi}{\partial z} = \begin{cases} \sum_{i=1}^{N} \omega_i A_i \frac{sinh(k_i(z+d))}{sinh(k_i d)} sin(k_i x - \omega_i t + \Phi_i) & z \leq \eta \\ 0 & z > \eta \end{cases} \tag{6}$$

from which the horizontal acceleration a_x and vertical acceleration a_z are derived as

$$a_x = -\frac{\partial^2 \phi}{\partial x \partial t} = \begin{cases} \sum_{i=1}^{N} \omega_i^2 A_i \frac{cosh(k_i(z+d))}{sinh(k_i d)} sin(k_i x - \omega_i t + \Phi_i) & z \leq \eta \\ 0 & z > \eta \end{cases} \tag{7}$$

$$a_z = -\frac{\partial^2 \phi}{\partial z \partial t} = \begin{cases} \sum_{i=1}^{N} \omega_i^2 A_i \frac{sinh(k_i(z+d))}{sinh(k_i d)} cos(k_i x - \omega_i t + \Phi_i) & z \leq \eta \\ 0 & z > \eta \end{cases} \tag{8}$$

For all wave conditions generated within the tank, an adequate approximation of the flow conditions at the velocimeter was achieved via the velocity field reconstruction. A summary of validation performance for the velocities, accelerations and free surface is shown in Table I. Reconstruction accuracies of $15 - 20\%$ were deemed acceptable, and around the order expected given the use and limitations of linear wave theory [4, 5].

TABLE I. AVERAGE NMSE ACHIEVED ACROSS ALL JONSWAP WAVES FOR THE WATER PARTICLE VELOCITIES U_X, U_Z AND ACCELERATIONS A_X, A_Z AT THE VELOCIMETER, AND THE FREE SURFACE η_2 AT WAVE GAUGE 2.

Variable	U_x	U_z	a_x	a_z	η_2
NMSE (%)	15.693	18.853	17.959	20.602	15.480

Where the reconstruction of the flow conditions at the velocimeter and free surface at wave gauge 2 could be compared with measured data, the measured flow conditions at the force collar were not available. However it should be emphasised that the flow conditions themselves are not the property of interest; the wave load is. The success of the velocity field reconstruction at the force collar shall therefore be determined not by the accuracy of the reconstructed flow conditions, but whether they provide a useful input to assist a wave loading prediction model. The steps to validate the constructed sum of linear waves indicate a satisfactory representation of conditions at the velocimeter. Whether this was repeated at the force collar will be in part determined by the performance of wave loading predictions models presented in the next section.

RESULTS

Models were implemented across a matrix of JONSWAP waves as a function of peak enhancement factor γ and peak frequency ω_p. Each model was trained on 300 data points and tested on an unseen set of 1000 data points. A purely data-based approach was first considered in which the wave gauge data is passed directly to the GP-NARX as an input, with the relationship between them learned. This is then compared with a physics-informed case of approximating the flow conditions at the force collar using linear wave theory for use as a GP-NARX input. A comparison of the performance for these cases across the range of wave states is shown in Figure 3.

The results motivate the use of machine learning and physical knowledge in combination, with the best performing model being the GP-NARX with physically informed inputs. The use of linear wave to estimate the water particle velocity and acceleration for use as inputs to the GP-NARX achieved a 15.93% average NMSE across all wave states, compared with 51.37% average NMSE when the measured free surface was passed directly to the GP-NARX. The physics-informed model was also able to outperform the purely data-based approach on all individual wave states, with even the poorest performing wave state (22.34% NMSE for a JONSWAP wave with $\gamma = 5.3$ and $\omega_p = 0.8Hz$) better the best performing purely data-based model wave state (36.55% NMSE for a JONSWAP wave with $\gamma = 5.3$ and $\omega_p = 0.7Hz$).

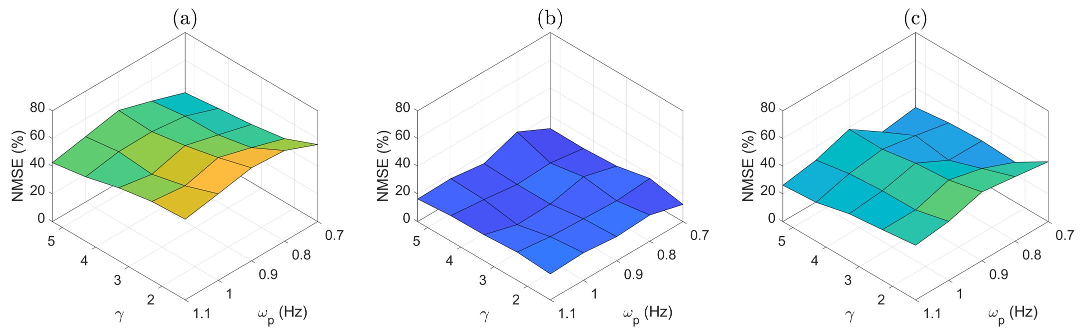

Figure 3: (a) NMSE surface of a GP-NARX with an SE kernel using wave gauge data as an input. (b) NMSE surface of a GP-NARX with an SE kernel as a function of approximated flow conditions. (c) The improvements in performance achieved as a result of using the approximated flow conditions as an input. Models were tested over a grid of JONSWAP waves as a function of peak enhancement factor γ and peak frequency ω_p.

Even though a GP-NARX is a flexible and effective data-based technique within a variety of applications, the inclusion of the reconstructed flow conditions as a model input was still able to improve prediction quality. By reducing the complexity of the learning task, the use of linear wave theory was able to assist the machine learning model, even though it only provides an approximation of flow conditions. This highlights perhaps the major finding of this work, that physical knowledge integrated within machine learning models does not have to be exact to be helpful. The limitations of linear wave theory are well understood [4, 5], and are unlikely to fully hold even within laboratory environments. This is observed within the $\sim 15-20\%$ errors in the velocities, accelerations and free surface within Table I. However, whether or not the approximated flow conditions were close to their true value, they were still able to assist the GP-NARX with wave loading prediction, which was their primary goal.

CONCLUSIONS

Novel wave loading prediction models were developed that utilised only incoming wave height as a model input. They were implemented on an experimental dataset of a monopile structure within a wave tank across a variety of representative ocean state spectra. The data from a single wave gauge, performing an equivalent role to wave radars installed on offshore structures, was used to predict a wave load measured by a force collar.

The use of linear wave theory and a GP-NARX in combination was able to offer significantly improved performance over a purely data-based approach, with the average NMSE across wave states falling from 51.37% to 15.93%. The approximation of flow conditions close to the monopile through physical knowledge provided an input to the GP-NARX that reduced the complexity of the learning task, allowing easier capture of functional structure and more effective utilisation of available data.

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