Ray Tracing Methodology for Elastic Wave Propagation Improve Simulation Applied to Aeronautic Composite Structures

FERNANDO SÁNCHEZ IGLESIAS, ANTONIO FERNÁNDEZ LÓPEZ and ALFREDO GÜEMES GORDO

ABSTRACT

Data processing and simulations for guided wave based Structural Health Monitoring (SHM) systems can be extremely challenging, especially in complex geometries. Aerospace structures, specifically, may contain integrated stiffeners, thickness changes and other structural details, which will result in a very complex signal on the sensor network due to the multiple reflection of the multiple modes on the structural elements. Currently, the most common methodologies to simulate Lamb waves are generally based in the finite element method (FEM), spectral element method (SEM). FEM models require a large amount of computational resources and therefore are limited in the size and number of cases that could be calculated, and SEM present some difficulties to implement out of plane reinforcements and to modelize structural details. For this reason, being able to perform a fast and efficient assessment of the propagation of elastic waves can be very helpful to reduce the need of physical tests, and could be used to identify the critical cases to run with a more detailed numerical simulation or to perform an stochastic analysis. In an attempt to solve this problem, this study presents an analytical methodology based on the ray tracing method, able to solve the propagation of symmetric and anti-symmetric elastic waves on a 2D space and able to take into account different material characteristics, propagation velocities and simple boundaries; the methodology considers different propagation modes independently and takes into account the splitting of rays and possible mode changes when encountering a boundary during the propagation.

The capabilities of this method are demonstrated on a series of analytical study cases and compared against physical tests, presenting an equivalent or better correlation than the finite element method, with a substantial reduction in the computational cost.

Antonio Ferna´ndez Lo´pez, ETSI Aerona´utica y del Espacio, Technical University of Madrid, Email: antonio.fernandez.lopez@upm.es.

INTRODUCTION

Due to the high cost of physical tests, numerical simulation can be especially helpful to predict the elastic wave propagation in light weight structures and may help improve the accuracy of these systems [1]. Computational numerical simulations provide a very effective way to capture and understand the structural behavior in terms of elastic wave response; general purpose computational codes such as time domain spectral finite element methods or explicit finite element methods are commonly used to calculate the wave propagation.

However, these codes require a large amount of computational resources and time, and may become a problem when trying to simulate large and complex structures. For these reasons, and also to increase the reliability of the simulations, there exists multiple different attempts of solving the problem without resorting to numerical simulation in the literature, analytically, via statistics or with artificial intelligence; some of these methods are able to achieve very good results, but the algorithms studied may be too complex to generalize or still require a spatial mesh, and therefore presenting similar limitations as the finite element method.

A different, very promising idea to solve the problem analytically, could be based in the ray tracing method. The ray tracing method is based on the assumption that the particle motion can be modeled as a number of idealized narrow beams (rays) which are advanced through the medium by discrete amounts. As they advance through the medium, these rays may interact with the medium itself or with different objects that may be present along their path.

Due to its simplicity in the implementation and computation, the ray tracing method has been used widely in scientific research for many different applications, most notably: astronomy, optical design, ocean acoustics or heat transfer. Specifically it has also been previously used in SHM to solve the guided wave wave propagation problem and it has proven its effectiveness [2, 3].

RAY TRACING METHODOLOGY

In general, elastic waves in solid materials are guided by the boundaries of the media in which they propagate. Waves in infinite metallic plates were among the first guided waves to be analyzed in 1917 by Horace Lamb, and have been extensively studied in literature since then for their applications in SHM. They present three distinctive propagation modes: Symmetric ($S0$), Anti-symmetric ($A0$) and Shear ($Sh0$) that can be treated as uncoupled for thin plate structures and therefore their propagation may be modeled with different rays. The shear mode generally carries the least amount of energy of the three in cases of active emission typical in SHM applications, and is the least sensitive to damages or events in light weight structures. Therefore, for simplicity of the method, it is not considered in the simulation.

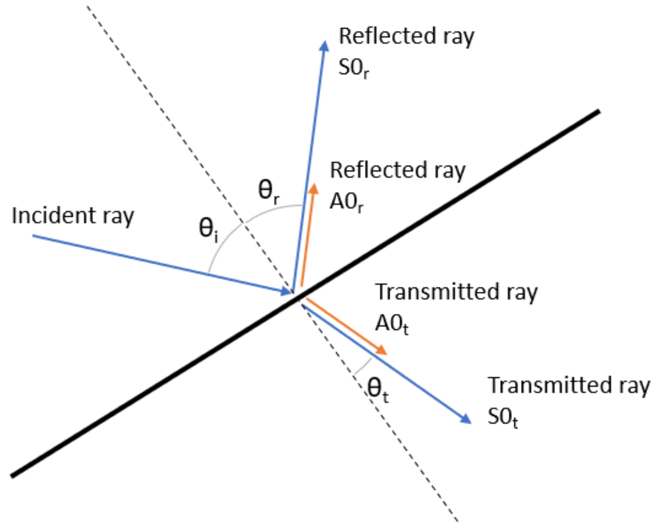

Figure 1. Ray splitting after encountering a boundary

When a ray encounters a boundary a component of the ray energy is reflected and another part is transmitted; the model takes into account the mode conversion phenomena [4] at each boundary and therefore, the transmitted and reflected component energy is divided between the different propagation modes. as shown in Figure 1., where $\theta_i = \theta_r$ and θ_t is estimated via the Snell's Law.

Ray propagation velocities are estimated via the dispersion curves of the S_0 and A_0 modes, obtained with the stiffness transfer matrix method STMM. As per [5], combining the momentum equation and the stress strain relation on a solid gives the following expression:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} \frac{\partial^2 u_l}{\partial x_k \partial x_j} \tag{1}$$

Considering harmonic motion, solutions of this equation can be found in terms of the bulk wave number ζ, the displacement amplitude $\vec{U}$ and the propagation direction $\vec{n}$ as:

$$u_i = U_i e^{j(\zeta n_j x_j - \omega t)} \tag{2}$$

Therefore, by taking $\lambda_{ijkl} = c_{ijkl}/\rho$, and introducing the phase velocity $v = \omega/\zeta$, the equation can be rewritten as follows:

$$\left(\lambda_{ijkl} n_k n_j - v^2 \delta_{il}\right) U_l = 0 => \left(\Lambda_{il} - v^2 \delta_{il}) U_l\right) = 0 \tag{3}$$

Where $\Lambda_{il} = \lambda_{ijkl} n_k n_j$. The previous equation represents an eigenvalue problem, that can be explicitly written as follows:

$$\begin{pmatrix} \Lambda_{11} - v^2 & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} - v^2 & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} - v^2 \end{pmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = 0 \tag{4}$$

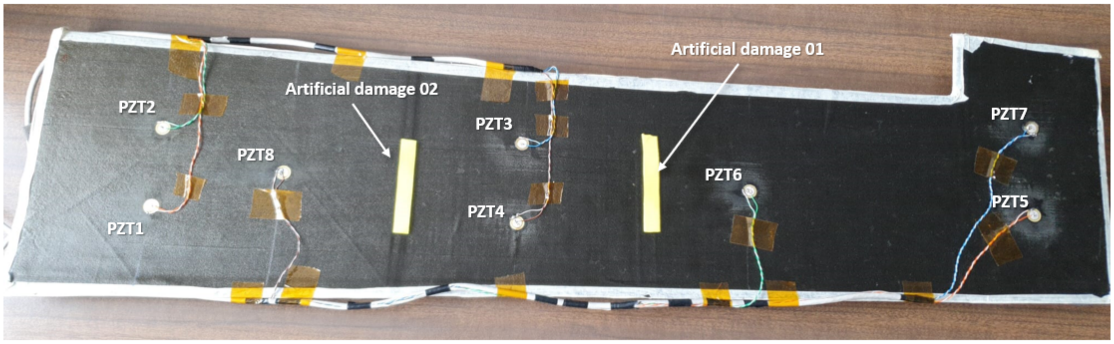

Figure 2. Case of study specimen used for the tests

From this equation the phase velocities can be obtained as the eigenvalues, and the U_l vectors are the eigenvectors. In general, the solutions of the problem will result in three phase velocities, that can be associated with the symmetric, antisymmetric and shear wave propagation modes for thin plates.

The stiffness transfer matrix method can be obtained by applying this equation on each layer of the laminate, and imposing the plane stress conditions on the free layers, as described in [6]; therefore being able to study the phase velocities for any arbitrary laminate.

CASE OF STUDY

The methodology is evaluated against a physical demonstrator of the front left wing lower cover of the remotely piloted aircraft system LIBIS, designed by the Technical University of Madrid.

The specimen is instrumented with an array of 8, 12mm piezoelectric (PZT) sensors, as shown in Figure 2, and the data is recorded with an Acellent SCANGENIE system, with a sampling frequency of 48M Hz.

As shown in the figure, artificial damages to emulate the effect of a possible delamination have been introduced in the specimen. Three sets of tests were performed considering the intact structure, the structure with the artificial damage 01 and the structure with both artificial damages. The results are compared with the simulation in the following section.

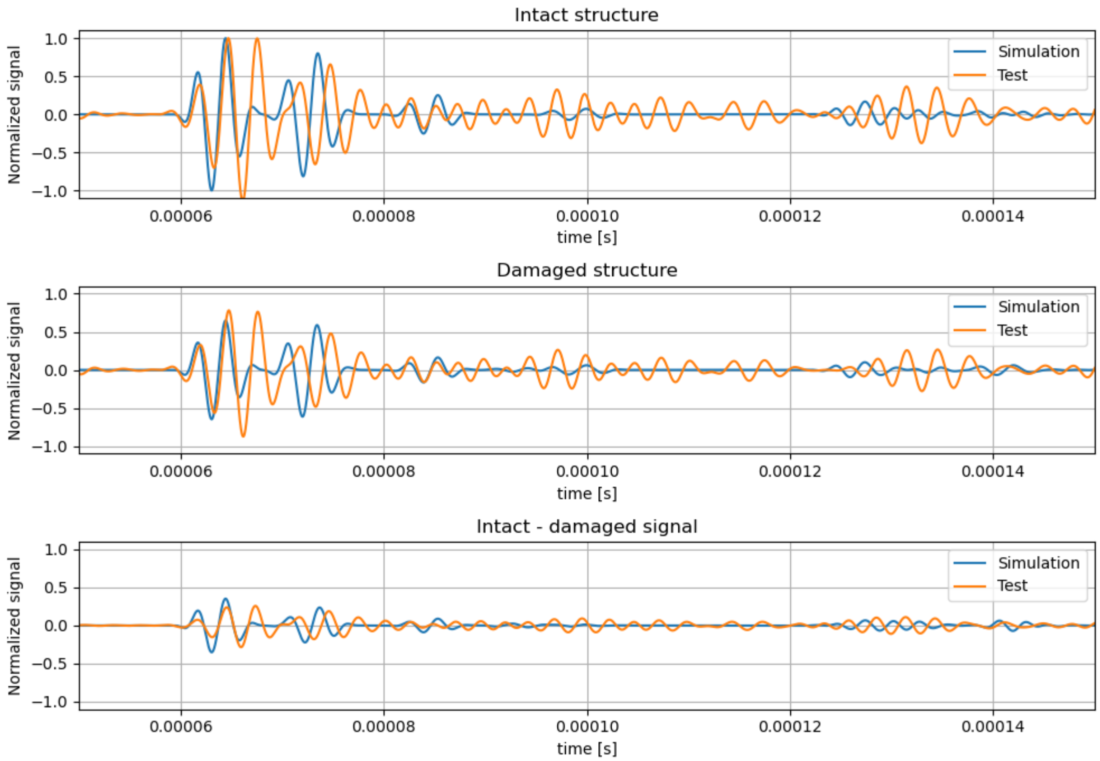

Figure 3. Result comparison of Path 8-6 with artificial damage 01

RESULTS AND DISCUSSION

The simulation is run with 301 initial rays originating from piezo-electric transducer $PZT8$. To simplify the analysis, the dispersion curves are calculated via the STMM method for one of the sections and then scaled by the thickness on the other two. The input signal used consist on a BURST3 at 350kHz; test results are presented as an average of 3 runs.

The results comparing the test and simulation signals are shown in Figure 3 for artificial damage 1 for the path between transducers $PZT8$ and $PZT6$ at both sides of the artificial damage. Both intact signals and scatter plots are also shown on the figure.

The energy absorption parameters of the damage are adjusted in order to match the amplitude of the first wave package arrival, with this values the results show an acceptable correlation on all sensors studied, as shown in the figure.

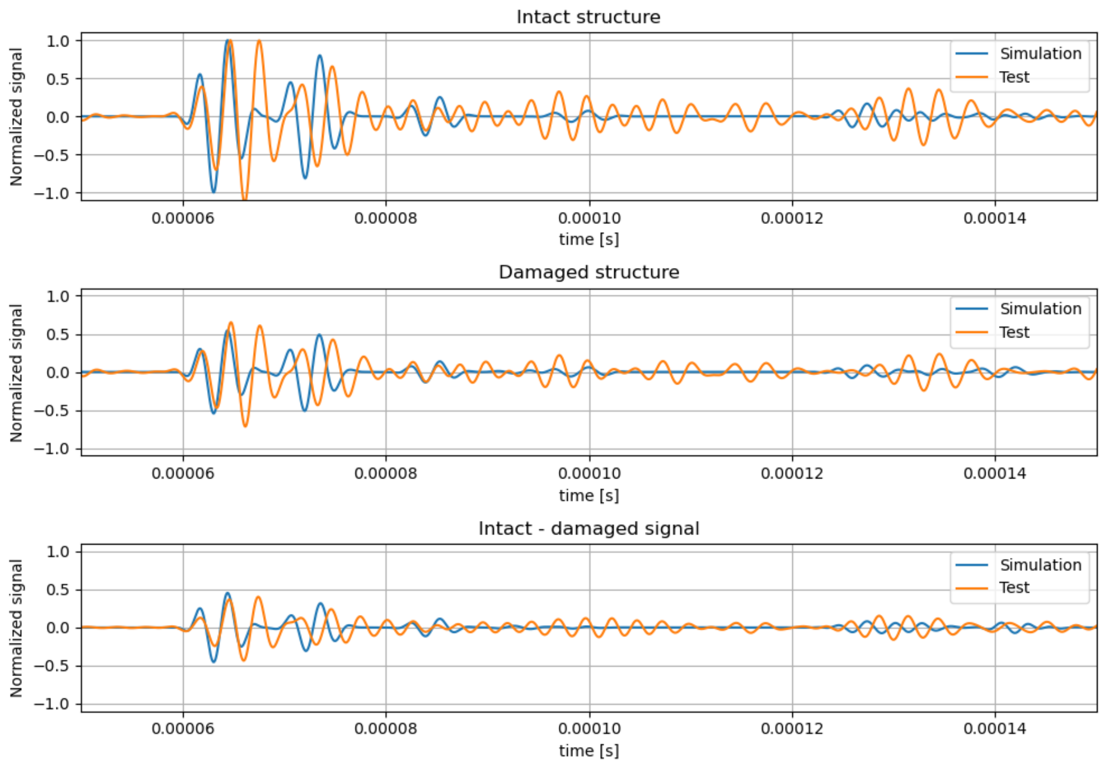

Figure 4. Result comparison of Path 8-6 with artificial damage 01 and 02

The results obtained for the case of artificial damage 1 and 2 are shown in Figure 4. The same parameters adjusted for the artificial damage 1 are applied on the damage 2 and the results still show a similar degree of correlation.

A visual representation of the elastic waves can be obtained by superimposing a rectangular grid over the ray propagation map. At a certain time instant the intensity of all rays under each point of the grid are added together generating an intensity plot. The results of this procedure are shown in Figure 5.

For this example, the grid has a side length of approximately $2mm$ and the model has been run with 2000 initial rays. The piezoelectric sensors are marked in red artificial damage is marked in blue in the figure.

CONCLUDING REMARKS

This article presents an efficient method to calculate the guided elastic wave propagation for a general 2D geometry, with an arbitrarily large number of boundaries or sensors. The method proposed consists on a computation of the propagation map and the extraction of the signal in the relevant sensors by superimposing all the intersecting rays in the sensors area.

The method is able to consider changes in material or structural details; different kinds of boundaries may be represented and its properties could be adjusted via FEM or experimental tests, and it has proven to accurately consider thickness changes and possible damages on a composite specimen, representative of a real aircraft structure.

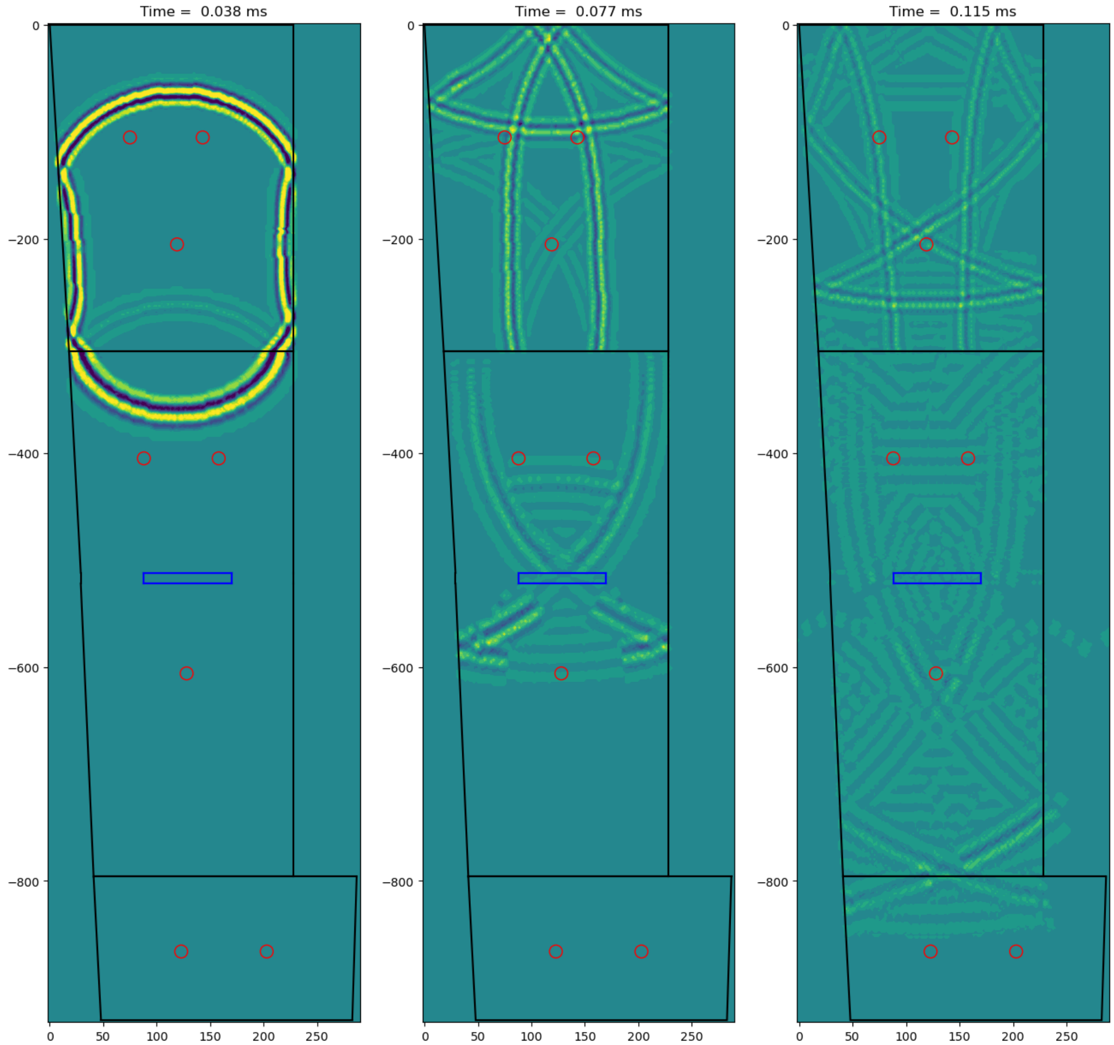

Figure 5. $S0$ Wave propagation color map at three relevant time points of the simulation

The implementation shown is able to solve moderately large size problems in a standard laptop without the need of high performance computing resources, which is a significant improvement when compared with other numerical approaches such as the finite element method that could require up to days of computation on specialized equipment.

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