Best Reconstruction Frequency of Lamb Waves Via Broadband Measurement

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ABSTRACT

This article presents the theoretical basis of the best reconstruction frequency (BRF) of Lamb waves. It has been proposed earlier by the author's group as the probing frequency in the refined time-reversal method for damage detection to achieve enhanced sensitivity and accuracy. It is shown to be the frequency at which the amplitude dispersion of the main mode of the reconstructed signal is minimum. It is then possible to compute the BRF from a single measurement of Lamb wave response under a broadband (e.g., Gaussian) excitation. Thus, it is no longer required to compute the similarity at different central frequencies of the narrowband input. It is further shown that the BRF is a system property of a given transducer-plate-adhesive system, which is independent of the distance between the transducers used for the time-reversal method in pitch-catch mode. This property makes it suitable for use when a network of sensing paths is used to identify damage locations.

INTRODUCTION

The difficulty in securing baseline signals for structures in their healthy state under every possible operating condition has driven the current surge in interest toward baseline signal-free methods of structural health monitoring (SHM). Even when baseline data are available, such a method would add to the reliability of the damage prediction. The time-reversal method (TRM) based on Lamb waves has been widely studied as a baseline-free method for SHM of plate-/shell-like structures [1–3]. In this method, the reconstructed signal obtained after a time-reversal process of the Lamb wave signal is compared with the original input waveform to detect the damage. However, unlike acoustic waves which inspired this process [4], the reconstruction of guided waves after the time-reversal process (TRP) would never be perfect due to amplitude dispersion. The time-reversibility, i.e., the similarity or correlation between the main mode of the reconstructed signal and the input signal for Lamb waves is in fact frequency dependent. This is why the sensitivity of the conventional TRM to the presence of damage has been found to vary from poor to good in the literature, raising questions about its use-

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fulness [5, 6]. The author's group has shown that the sensitivity of the damage index to damage is maximum and significantly higher if the probing Lamb wave is excited at the frequency when the time-reversibility in the undamaged state is maximum over a given range of frequencies [7, 8]. Termed the best reconstruction frequency (BRF), this probing frequency and the use of an extended wave packet of the reconstructed signal instead of only the main mode has led to a large improvement in the sensitivity and localization accuracy of the refined time-reversal method (RTRM) proposed earlier [9]. However, the definition of the BRF has been phenomenological so far, which was determined by actually computing the similarity or correlation coefficient of the reconstructed and input waveforms at various excitation frequencies of a narrowband Hann window-modulated tone burst signal over a desired frequency range.

In this article, the theoretical basis of the BRF is presented. It is shown to be the frequency at which the amplitude dispersion of the main mode of the reconstructed signal is minimum. It will occur when the slope of the main mode amplitude versus frequency curve is zero. It is possible to obtain the main mode amplitude versus frequency curve for a transducer-structure system from a single measurement of Lamb wave response under a broadband (e.g. Gaussian) excitation covering the desired maximum probing frequency. The BRF can then be obtained as the frequency at which the slope of this curve is zero and its curvature is minimum. Thus, it is no longer required to compute the similarity at different central frequencies of the narrowband input. It is further shown that the BRF is a system property of a given transducer-plate-adhesive system, which is independent of the distance between the transducers used for the TRM in pitch-catch mode. This property makes it suitable for use when a network of sensing paths is used to identify damage locations. The proposed method of computing the BRF is demonstrated through experiments conducted on a 3 mm thick aluminium plate using square-shaped piezoelectric patches as transducers for generating and sensing the guided wave propagating through the plate.

BEST RECONSTRUCTION FREQUENCY: THE PHENOMENOLOGICAL DEFINITION

Let us consider a sensing path consisting of two piezoelectric wafer active sensor (PWAS) transducers pasted in the pitch-catch configuration on a plate/shell-type host structure. In the TRP [4], one of them, say PWAS-A, is actuated with a voltage signal to generate Lamb waves in the host structure. When the Lamb wave reaches the other transducer, say PWAS-B, it induces an voltage signal, which we call the forward response. The forward response is reversed in time and then reemitted from PWAS-B to the source transducer PWAS-A. When there is no damage between the two transducers, the signal received at the source transducer would have a central main mode accompanied by a side lobe on its either side. The normalized main mode should ideally match the normalized input signal in the case of no damage. Hence the signal at the PWAS-A obtained after the TRP is called the reconstructed signal.

When there is a damage between the two transducers, the normalized main mode of the reconstructed signal ϕ_{rc} would no longer match the input signal. The mismatch between the main mode and the input signal ϕ_a (both normalized) is often quantified by a damage index (DI) based on the L_2 norm error as follows:

$$DI_{L_2} = \sqrt{\int_{t_i}^{t_f} [\phi_{rc}(t) - \phi_a(t)]^2 dt \Big/ \int_{t_i}^{t_f} \phi_a^2(t) dt} \tag{1}$$

where t_i and t_f denotes the starting and end time instances of the signals under comparison. Their values for comparing the main mode can be obtained from

$$t_i = t_m - t_{sl}/2, \qquad t_f = t_m + t_{sl}/2$$

where t_m and t_{sl} denotes the time instances of the peak of the main mode and the width of the input signal, respectively. The percent similarity between the two signals is given by $100(1 - DI_{L_2})$. So, $DI = 0$ would mean a perfect match, which would ideally indicate no damage and its high value would mean a significant mismatch indicating presence of damage.

However, due to amplitude dispersion of Lamb waves, the reconstruction is never perfect even in healthy structure. Thus, the threshold DI corresponding to the healthy state of the structure is not zero. Indeed, its value would depend on the probing frequency and can be rather high, severely reducing the sensitivity of the DI to the presence of damage and hence the effectiveness of the TRM for damage detection.

To address this issue, Agrahari and Kapuria [7, 8] proposed to probe the structure at what they termed the *best reconstruction frequency*, which was defined as the frequency at which the similarity of the reconstructed signal's main mode with the input signal is maximum over a desired range of excitation frequency. This frequency was determined by actually computing the similarity at different frequencies over the desired range. It is, however, a phenomenological description of the BRF. The following section provides a theoretical basis of the BRF.

BEST RECONSTRUCTION FREQUENCY: THE THEORETICAL BASIS

For circular crested waves in plates, the forward response $\phi_s(\omega)$ due to a harmonic excitation $\phi_a(\omega)$ of frequency ω applied at PWAS-A can be expressed in terms of a transfer function $H(\omega)$ as

$$\phi_s(\omega) = H(\omega)\phi_a(\omega) \tag{2}$$

with

$$H(\omega) = \frac{1}{\sqrt{d}}[S(\omega)e^{ik^{S_0}d} + A(\omega)e^{ik^{A_0}d}] = H_S(\omega) + H_A(\omega) \tag{3}$$

In the above equation, we have considered two fundamental Lamb wave modes S_0 and A_0 that are usually employed in SHM applications. k^{S_0} and k^{A_0} are the wavenumbers corresponding to these modes, while H_S and H_A denote the respective transfer functions. The reconstructed signal can now be obtained as

$$\phi_{rc}(\omega) = H(\omega)H^*(\omega)\phi_a^*(\omega) = H_{rc}(\omega)\phi_a^*(\omega) \tag{4}$$

where the superscript * means complex conjugate. Substituting Eq. (3) into Eq. (4), the amplitude of the main mode of the reconstructed signal can be obtained as

$$M(\omega) = |H_S(\omega)|^2 + |H_A(\omega)|^2 = \frac{1}{d}(|S(\omega)|^2 + |A(\omega)|^2) \tag{5}$$

The similarity of the main mode would be maximum when the dispersion of its amplitude $M(\omega)$ would be minimum. The amplitude dispersion would be minimum at a frequency at which $\frac{\mathrm{d}M}{\mathrm{d}\omega} = 0$ and $|\frac{\mathrm{d}^2M}{\mathrm{d}\omega^2}|$ is minimum. As the normalized $M(\omega)$ is independent of d, so will be the BRF. Thus, for a network of transducers (of same type, size, and adhesive), the BRF would be the same for all sensing paths irrespective of the path lengths. It makes the BRF a system property, which can be determined theoretically or through measurements.

The computation of $M(\omega)$ requires the individual transfer functions $H_S(\omega)$ and $H_A(\omega)$ for S_0 and A_0 modes. They can be determined by separating $\phi_s^S(t)$ and $\phi_s^A(t)$ corresponding to S_0 and A_0 mode waveforms, respectively, from the forward time history response $\phi_s(t)$. $H_S(\omega)$ and $H_A(\omega)$ can then be obtained as

$$H_S(\omega) = \phi_s^S(\omega)/\phi_a(\omega), \quad H_S(\omega) = \phi_s^A(\omega)/\phi_a(\omega) \tag{6}$$

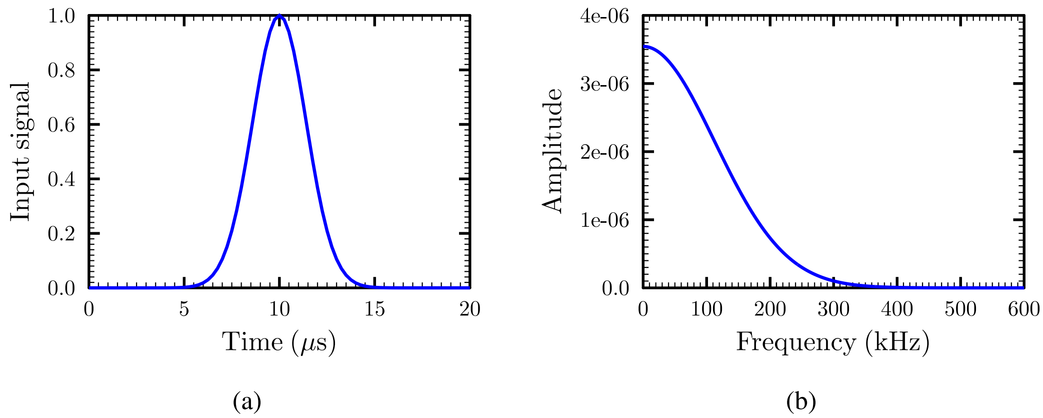
where $\phi_s^S(\omega)$, $\phi_s^A(\omega)$, and $\phi_a(\omega)$ are the Fourier transforms of $\phi_s^S(t)$, $\phi_s^A(t)$, and $\phi_a(t)$, respectively.

If a narrowband input like the widely used Hann window modulated sinusoidal pulse is used as excitation, the above transfer functions will be valid only for a frequency band ranging from a non-zero lower limit to an upper limit. This would require multiple measurements at different central frequencies to cover the desired range of frequency. As a better alternative, the transfer functions can be determined for frequency ranging from zero to any desired value from just one measurement by choosing a broadband excitation such as Gaussian pulse [9]. This process will directly yield the BRF from a single measurement of forward response in a sensing path without requiring to actually perform the TRP and compute the similarity of reconstructed signal at different excitation frequencies.

EXPERIMENTAL RESULTS AND DISCUSSION

An aluminium plate of 1500 mm $\times$ 1200 mm $\times$ 3 mm size was considered for the experimental determination of the BRF using the proposed method. Two square-shaped PZT wafers of SP-5H type (Sparkler Ceramics, India) of 10 mm length and 0.25 mm thickness were used as transducers PWAS-A and PWAS-B to actuate and sense Lamb wave signals. They were pasted on the plate using a two-component epoxy adhesive at a centre to centre distance of 300 mm. The adhesive thickness was measured with the help of a laser displacement sensor as 50 $\pm 5\mu$m. The voltage actuation and sensing at the PWAS transducers were performed using the high-frequency signal generator cum data acquisition system with inbuilt charge amplifiers and associated pre- and post-processing software made by Quazar Technologies Pvt. Ltd, India. The analog-to-digital (ADC) and DAC conversion resolution was 14 bit and a sampling frequency of 72 mega samples per second (Msps) was used for the data acquisition. The number of 20 repetitive measurements to be averaged for each signal data was set to 20 or more to improve the signal-to-noise ratio of the output signal.

PWAS-A was actuated with a broadband Gaussian pulse voltage signal $V(t) = V_0 \exp\left[-(\frac{t-t_0}{\tau})^2\right]$ with $V_0 = 20$ V, $t_0 = 10$ μs and $\tau = 2$ μs (Fig. 1 (a)). Unlike the



(a) (b)

Figure 1. (a) Broadband Gaussian pulse with $\tau = 2$ μs. (b) Its frequency spectrum

narrowband modulated tone burst signals (generally used for excitation), its frequency spectrum ranges from 0 to a certain frequency (400 kHz in this case) (Fig. 1 (b)).

Figure 2 plots the sensor output showing the S_0 mode, A_0 mode, and edge reflections. The A_0 mode waveform gets mixed with the edge reflections for a small portion towards its end. Although this can be avoided by changing the spacing between the transducers or by eliminating the edge reflections by putting some damping material (e.g. play dough) at the edges, we take the output till t_{A_0} shown in the figure for obtaining the transfer function. The sensor output up to t_{A_0} was split at t_{S_0} to obtain the S_0 and A_0 mode response signals, which in turn were used to obtain the transfer functions H_S and H_A, respectively. Finally, the TRP transfer function of the main mode of the reconstructed signal was computed as $M(\omega) = |H_S|^2 + |H_A|^2$. Figure 3 show that there are some small undulations in H_A, which are clearly because the A_0 mode was cut a little short at t_{A_0}, but it does not affect the overall shapes of the respective transfer functions, particularly the location of the peak with lesser curvature, which concerns the best reconstruction frequency. The peak occurs at 112 kHz, which should be the best reconstruction frequency for this plate-adhesive-PWAS system.

To verify the predicted BRF, we obtain the BRF by using the earlier method of finding the frequency corresponding to maximum similarity of the reconstructed signal with input signal, both normalized. The percent similarity is computed for reconstructed signals obtained with narrowband Hann window modulated sinusoidal tone burst excitations of five count with central frequency ranging from 70 kHz to 300 kHz. The similarity versus central frequency plot presented in Fig. 4 shows that the maximum similarity occurs at 110 kHz, which matches the BRF (112 kHz) predicted as per the newly proposed definition very well.

CONCLUDING REMARKS

A theoretical basis has been presented for the best reconstruction frequency (BRF) of Lamb waves in a host plate-PWAS transducer-adhesive system, which was defined earlier in a phenomenological perspective. It is shown to be the frequency at which

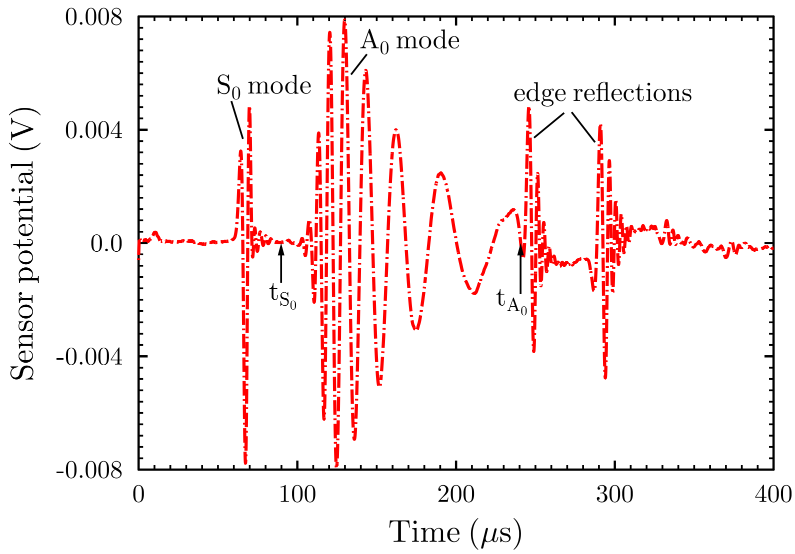


Figure 2. Sensor potential under broadband Gaussian pulse excitation

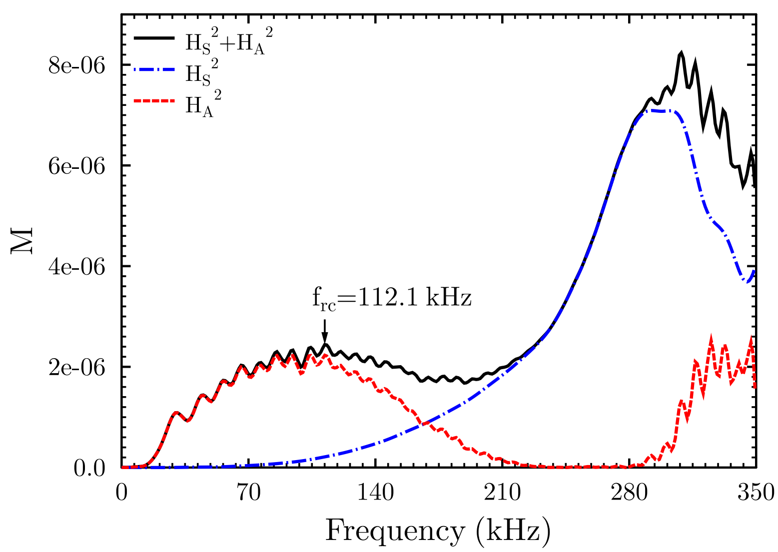


Figure 3. Best reconstruction frequency prediction from main mode transfer functions of S_0 and A_0 modes of broadband response

the amplitude dispersion of the main mode of the reconstructed signal is minimum. A methodology has been proposed based on this definition to compute the BRF from a single measurement of Lamb wave response under a broadband excitation, without actually computing the similarity at different central frequencies of a narrowband input.

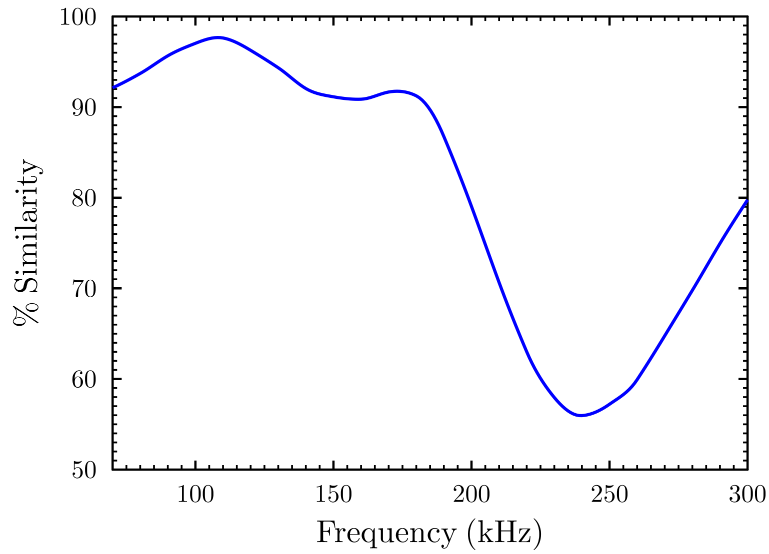


Figure 4. Similarity of the reconstructed signals obtained with Hann window modulated tone burst excitations of varying central frequency

The proposed method is demonstrated through experiments conducted on an aluminium plate using square-shaped PWAS actuator and sensor. The predicted BRF from a single measurement using a Gaussian excitation signal match excellently well with the result obtained from the earlier phenomenological method based on actual similarity calculations at multiple frequencies.

Further, the BRF is shown to a system property of a given transducer-plate-adhesive system, which is independent of the distance between the transducers used for the TRP in pitch-catch mode. This property makes it suitable for use when a network of sensing paths is used to identify damage locations.

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