Sensor Location Optimization for Efficient Structural Health Monitoring Through Intelligent Design

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ABSTRACT

Vibration based Structural health monitoring (SHM) is an emerging field in Civil engineering, which evaluates the structure's integrity and performance. It facilitates the early identification of damage/deterioration and reduces maintenance costs. Due to technological development in research nowadays, sensors play a critical role in SHM, by offering consistent and reliable measurements of structural dynamic parameters, such as displacement, temperature, strain, and vibration, thereby enabling timely detection of potential issues, targeted maintenance, and repairs. However, choosing the correct sensor locations on a structure can be ambiguous, hence, the development of an effective sensor optimization technique is essential so that it can reduce the maintenance cost. This involves identifying the most relevant parameters to monitor and determining the optimal number and location of sensors. Too many sensors can result in excessive data and processing requirements, leading to increased costs and reduced reliability. Conversely, too few sensors can result in incomplete data and reduced accuracy, leading to missed or delayed detection of potential problems. Hence, sensor optimization is necessary to ensure optimal data acquisition and efficient monitoring. The algorithm proposed in this study for sensor optimization employs Particle Swarm Optimization (PSO), Bayesian Optimization (BO), and Optuna to determine the optimal sensor locations for monitoring a structure. To achieve this, the algorithm employs modal assurance criteria (MAC), Fisher Information Matrix (FIM), and both MAC and FIM as objective functions for each of the optimization algorithms in addressing the Optimal Sensor Location Problem (OSLP). The algorithm aims to provide an efficient and effective method for determining the best sensor locations for monitoring the structure's health and identifying system parameters, such as frequencies, mode shapes, and damping ratios. The algorithm aims to provide a reliable method for identifying system parameters such as mode shapes, frequencies, and damping ratios by finding the best sensor locations. By leveraging the strengths of PSO, BO, and Optuna, the algorithm optimizes the objective function to identify the best sensor locations for the structure and also gives minimum and maximum number of sensor locations for identifying the system parameters. Based on the optimized results obtained from utilizing the optimization techniques in the OSLP, it can be concluded that the optimal sensor locations can guarantee improved linear independence of the MSV and are validated with experimental modal parameters. Furthermore, the proposed algorithm can provide a reliable and efficient method for determining the best sensor locations to monitor a structure. Such findings can greatly benefit the field of structural health monitoring and contribute to enhanced safety and reliability of engineering structures.

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INTRODUCTION

Structural Health Monitoring (SHM) is an important process for ensuring the safety and reliability of critical infrastructure. In SHM, sensors play a vital role in providing real-time data on various aspects of the structure, including strain, acceleration, displacement, temperature, and humidity, which help in detecting changes in structural behavior. Continuously monitoring a structure's integrity and performance can help detect and address potential issues before they cause harm. Modal analysis is one of the most crucial steps in analyzing structural health of infrastructure.

Modal analysis is a technique used to understand the dynamic behavior of structures under different loading conditions. Experimental modal testing is used to identify modal parameters, such as natural frequencies and mode shapes. Accurate estimation of modal parameters relies on factors such as sensor quantity, sensor placement, excitation force, and data acquisition system. Sensor placement is particularly critical for optimal modal parameter estimation. The Optimal Sensor Location Problem (OSLP) seeks to find the optimal locations for sensors that can provide the sufficient information to estimate the modal parameters of a structure accurately. A secondary objective of OSLP is to minimize the number of sensors required while ensuring that the chosen locations are redundant and sufficient to provide a good estimation of the modal parameters. By using optimization techniques, we can efficiently search for the Optimal Sensor Locations (OSL) and minimize the number of sensors required to monitor the structure.

To solve the OSLP, selecting an efficient criterion and appropriate optimization method is crucial. The objective is to find the minimum number of sensor locations that can provide adequate information on the modal parameters of the structure [1]. This paper uses Modal Assurance Criterion (MAC) and Fisher Information Matrix (FIM) as criteria to solve the OSLP . MAC is used as an objective function to optimize the sensor locations by selecting locations that have a high degree of linear independence [2, 3], while FIM is used because it provides a way to quantify the information content of measurement data in estimating the modal parameters of a structure [1–3]. Using this information, it is possible to identify the OSL that gives comprehensive structural health information while minimizing the number of sensors required. Additionally, we use a combination of MAC and FIM as the objective function to converge at the best possible location for a given use case.

In this paper, the following optimization techniques have been employed; Particle Swarm Optimization (PSO) a stochastic optimization method that can be applied to a wide range of optimization problems, including OSLP [3–5], Bayesian Optimization (BO) another popular optimization method that uses Bayesian inference to optimize a function which is particularly effective when the objective function requires a limited number of evaluations [1, 6, 7], and Optuna a Python library for hyper-parameter optimization that is designed for machine learning tasks, but it can also be used to solve optimization problems [6] as optimization techniques to find the optimal values for each case while using the mentioned criterion as objective functions [1, 4–8]. By using these optimization techniques, we can efficiently explore the locations for sensors and reduce the number of required sensors for effective structural monitoring. Finally, the results obtained using these optimization techniques are tested on a real structure and are compared to determine the best solution.

RELATED WORK

This paper presents a study on solving the OSLP whose objective is to identify the optimal sensor placements on a structure, aiming to enhance the effectiveness of health monitoring for SHM using an ensemble of optimization techniques. Several studies have proposed different optimization techniques for solving the OSLP problem. Meo and Zumpano [2] presented an approach for finding OSL on a bridge structure. Yi et al. [9] proposed method to tackle the OSLP for high-rise buildings based on genetic algorithms, while Qin and Lin [3] used PSO mainly. Other studies have also employed novel optimization algorithms, such as the novel PSO algorithm proposed by Zhang and Xing [4], and the PSO algorithm developed by Kennedy and Eberhart [5]. BO has also been previously used in optimizing sensor placement, as demonstrated by Frazier [7] and the Optuna framework developed by Akiba and Sano [6]. Yi et al. [9] combined multiple optimization strategies to optimize sensor placement, while Li et al. [1] used a hybrid genetic algorithm and BO method to tackle the OSLP. While there have been numerous studies on optimal sensor placement for SHM, there is still a gap in research. One of the main reasons for the gap is the complexity and non-linearity of the problem. To the best of our knowledge, no study has explored the effectiveness of combining multiple optimization techniques for OSLP. In this study, we present an ensemble approach that combines different optimization techniques to solve the OSLP problem which takes advantage of the strengths of each individual optimization technique and combines them to achieve a more versatile and effective solution.

METHODOLOGY

The aim of this study is to select the OSL while minimizing the number of sensors required. The methodology proposed in this study provides a systematic and efficient approach for selecting the optimal sensor location to monitor structural health involving three key steps. In this methodology, firstly, modal analysis is performed on the structure to identify the natural frequencies and mode shapes which helps us in identifying areas of high stress or displacement in the structure. MAC and FIM are then employed as criteria for OSLP to find locations that are critical in identifying mode shapes. Secondly, optimization techniques such as BO, PSO, and Optuna are applied to identify the OSL. Finally, MAC, FIM, and the combination of both MAC and FIM were used as objective functions for each of the above optimization techniques to locate integral and critical positions on the structure that capture the highest degree of damage and/or change in behaviour of the structure.

Modal Analysis

A modal analysis of the structure is conducted before OSLP is implemented to gain insight into its dynamic characteristics. This study utilized ANSYS software for the modal analysis of structures. Structures or systems are represented as interconnected masses and springs in modal analysis. The natural frequencies indicate the frequencies at which the structure or system will vibrate without any external stimulus. The mode shapes depict the deformation or vibration patterns linked to each natural frequency. The effectiveness of optimization techniques depends on the establishment of appropriate objective functions and coding methods.

The study collected the data by employing the ANSYS 18.1 software to develop a finite element model (FEM) of a Cantilever beam. The objective was to identify the most suitable positions for acceleration sensors on the beam in a single direction. The FEM utilized 1000 potential nodes and 1000 potential degrees of freedom (DOFs) to generate the required data for modal analysis.

Criteria for OSLP

In this paper, for evaluating the suitability of sensor locations in modal analysis using a FEM model. The utilization of MAC, FIM, and MAC and FIM is employed for evaluating the correlation between the mode shape vectors(MSV) at the designated measurement coordinates. MAC and FIM both are measures of the quality of the sensor locations. MAC quantifies how well the mode shapes are correlated with each other, while FIM measures the level of sensitivity of the estimated mode shape to variations in the positions of the sensors. By optimizing the sensor locations based on both MAC and FIM, we can ensure that the mode shapes are well-resolved, robust to noise, and accurately estimate the modal parameters of the system.

Once the mode shapes have been obtained from ANSYS, the subsequent task is to determine the OSL. This process entails picking a subset of nodes from a set of n nodes in the FEM. Each node i in the set of n nodes has k_i degrees of freedom (DOF) in total, forming a DOF set X consisting of a total of $n = k_1 + k_2 + k_3 + \ldots + k_n$ DOFs. To identify the OSL, one or more criteria are utilized to select s DOFs, denoted as $y_1 + y_2 + y_3 + \ldots + y_s$, from the set X. Subsequently, the accuracy of the selected DOFs is assessed using techniques such as the MAC or FIM. If the selected DOFs exhibit an optimal quantitative value based on the chosen criterion, they are considered the optimal locations for s sensors.

The MAC method evaluates the correlation between two sets of mode shapes. It is computationally efficient and suitable for analyzing large-scale structures, as it only requires the mode shapes obtained from experimental testing or finite element analysis. The MAC values are calculated using the following expression:

$$\text{MAC}_{ij} = \frac{(\phi_{si}^T \phi_{sj})^2}{(\phi_{si}^T \phi_{si})(\phi_{sj}^T \phi_{sj})} \tag{1}$$

Here, ϕ_{si} and ϕ_{sj} represent the ith and jth MSV, obtained from the FEM, respectively. Only the selected degrees of freedom are present in these vectors. The values of the non-diagonal terms of the MAC matrix indicate the independence of the mode shapes and thus, sensor locations. The smaller the value the higher the independence of the mode shapes. Therefore, the non-diagonal terms of the MAC matrix are used as a criterion for evaluating the suitability of sensor locations.

FIM considers how sensitively the system responds to each structural parameter, allowing sensors to be placed where they will yield the most useful data for model updating and system identification. FIM as an OSLP criterion has the benefit of offering a more precise and complete solution than other criteria like the MAC. Moreover, FIM can handle closely spaced modes that are challenging for MAC as it accounts for the covariance between different parameters and can effectively differentiate between closely spaced modes. However, it may not be effective in identifying the modes that are poorly excited or have low energy content. The selection of optimal sensor locations is achieved by maximizing the determinant of the FIM, which is evaluated using the Frobenius norm. The FIM is calculated as:

$$J = E[(q - \hat{q})(q - \hat{q})^T] = [\frac{(\phi_s^T \phi_s)}{\sigma^2}]^-1 = Q^-1 \quad (2)$$

Here, ϕ_s represents the matrix of MSV obtained from the FEM, while the covariance matrix σ corresponds to the noise inherent in the measurements. The determinant of the FIM is maximized by selecting candidate sensor locations that lead to the largest Frobenius norm of the FIM, given by:

$$\|Q\|_F = \|\phi_s^T \phi_s\|_F \quad (3)$$

In this study, we employed a combination of optimization techniques i.e, PSO, BO, and Optuna, within an ensemble model to address the OSLP problem. Our objective was to optimize sensor locations, utilizing MAC, FIM, and MAC and FIM as objective functions.

Optimization Algorithms:

PSO is a heuristic optimization algorithm inspired by the flocking behavior of birds. It aims to solve optimization problems by utilizing a set of particles. Each particle represents a potential solution and is initialized with a position and velocity in the problem's search space. The PSO algorithm iteratively updates the positions and velocities of the particles to search for the optimal solution.

Let's consider a D-dimensional problem space, where each particle's position is denoted by $\mathbf{X}_i = (X_{i1}, X_{i2}, \ldots, X_{iD})$. Here, $X_{i,d}$ represents the value of the dth dimension for the ith particle. The bounds for each dimension are denoted as l_d and u_d, where $d \in [1, D]$.

The velocity of a particle is denoted by $V_i = (V_{i1}, V_{i2}, \ldots, V_{iD})$, which controls the particle's flight direction and distance. The velocity is constrained by maximum ($V_{\max}$) and minimum ($V_{\min}$) values. Each particle also maintains its personal best position $P_i = (P_{i1}, P_{i2}, \ldots, P_{iD})$ and the global best position of the entire swarm $P_g = (P_{g1}, P_{g2}, \ldots, P_{gD})$.

The PSO algorithm can be summarized by the following equations:

Velocity update equation:

$$V_{id}(t+1) = w \times V_{id}(t) + c_1 \times \text{rand1} \times (P_{id} - X_{id}(t)) + c_2 \times \text{rand2} \times (P_g - X_{id}(t)) \quad (4)$$

Here, $V_{id}(t+1)$ represents the updated velocity of the particle at time $t+1$, $V_{id}(t)$ is the velocity of the particle at time t, w is the inertia weight, c_1 and c_2 are acceleration coefficients, rand1 and rand2 are random numbers between 0 and 1. P_{id} is the personal best position of the particle, and $X_{id}(t)$ represents the position of the particle at time t. P_g is the global best position of the swarm.

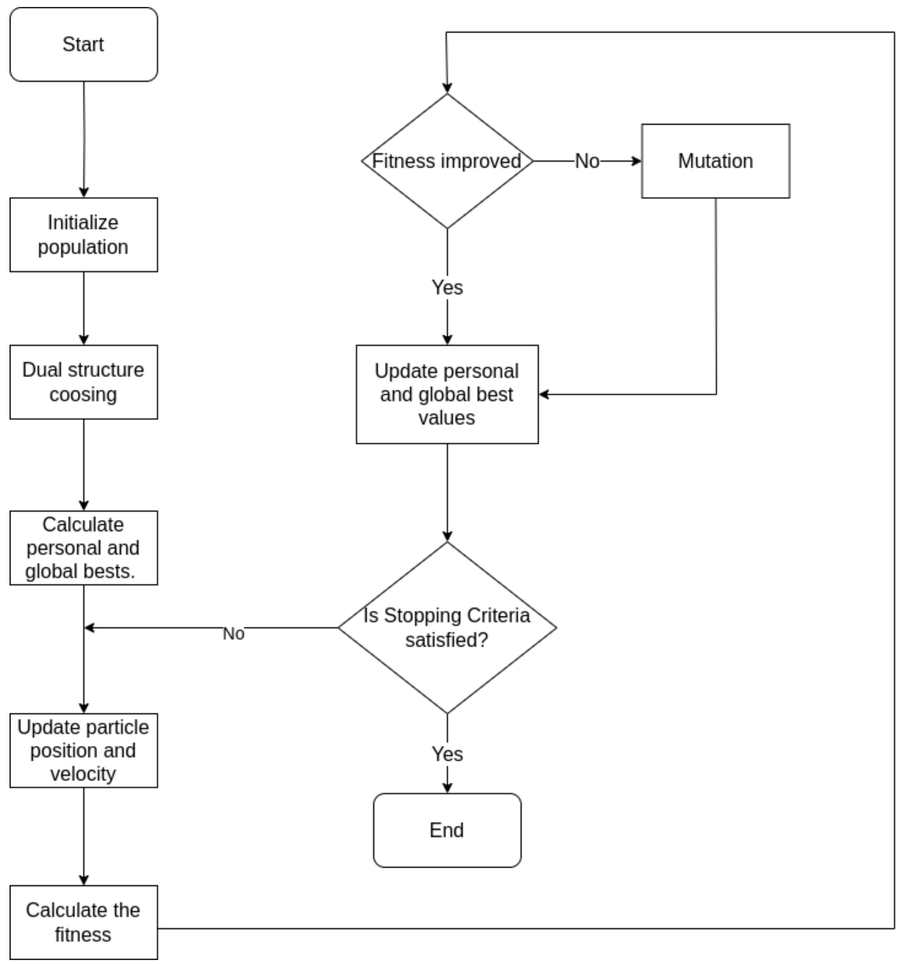


Figure 1. Flowchart depicting the functionality PSO

Position update equation:

$$X_{i,d}(t+1) = X_{i,d}(t) + V_{i,d}(t+1) \tag{5}$$

In the position update equation, $X_{i,d}(t+1)$ denotes the updated position of the particle at time $t+1$, $X_{i,d}(t)$ is the position of the particle at time t, and $V_{i,d}(t+1)$ is the velocity of the particle at time $t+1$.

The PSO algorithm proceeds continually, updating each particle's locations and velocities, until the best solution is discovered or a certain number of iterations is reached. Since the positions of the particles are constrained to integers, the velocities calculated at each iteration are rounded off to the nearest integer.

BO is a technique used to optimize expensive black-box functions that do not have a known analytical form. The goal of BO is to find the input values that minimize (or maximize) the output of the black-box function. The implementation of BO involves several steps. First, a Gaussian Process (GP) model is fit to the data points observed so far, where the input values are the independent variables, and the output values are the dependent variables. The GP model is used to model the unknown black-box function and estimate the distribution of the function values at unobserved points. The mean function of the GP model is often denoted by $\mu(x)$, and the covariance function by $k(x, x')$. The acquisition function, $\alpha(x)$, is defined as the expected improvement of the function value at a new input point, x, over the current best function value, f_{min}. The acquisition function is often chosen to balance exploration and exploitation of the function landscape and can be formulated as:

$$\alpha(x) = E\left[\max(f_{\min} - f(x), 0) \mid x, D\right] \tag{6}$$

where f(x) is the unknown black-box function, D is the set of observed data points, and $E[]$ is the expectation operator. Then, the input values that maximize the acquisition function are chosen as the next set of points to evaluate the black-box function. The black-box function is evaluated at these points, and the GP model is updated with the new data points using the Bayesian updating rule:

$$p(f \mid x, y, D) \propto p(y \mid f, x, D), p(f \mid x, D) \tag{7}$$

where $p(y \mid f, x, D)$ is the posterior distribution of the function given the observed data, y, $p(y \mid f, x, D)$ is the likelihood of the data given the function and inputs, and $p(f \mid x, D)$ is the prior distribution of the function given the inputs. This process of choosing new points to evaluate the black-box function based on the GP model and the acquisition function is repeated until a stopping criterion is met, such as a maximum number of iterations or a minimum improvement in the function value. Overall, the implementation of BO involves fitting a GP model to the data, defining an acquisition function based on the GP model, selecting new points to evaluate the black-box function based on the acquisition function, and updating the GP model with the new data. This process is repeated iteratively until the optimal input values are found.

Optuna is a library used for hyperparameter optimization. Optuna aims to minimize the time and resources needed to identify the best hyperparameters for a specific model and dataset. The implementation of Optuna involves defining a search space for the hyperparameters of the model. The search space can be defined using either a set of discrete values, a continuous range, or a combination of both. Optuna then searches the space of hyperparameters using various optimization algorithms. The Tree-structured Parzen Estimator (TPE) algorithm uses two probability density functions, one for the objective function and another for the hyperparameters and selects the hyperparameters that maximize the ratio of these probability densities. The formula for the ratio of probability densities is:

$$\frac{p(x \mid y = 1)}{p(x \mid y = 0)} \tag{8}$$

where x is a hyperparameter configuration, y is a binary variable that indicates whether the objective function value is above or below a threshold, and $p(x \mid y = 1)$ and $p(x \mid y = 0)$ are the probability densities of x given that y is 1 or 0, respectively. To use Optuna, the user first defines the objective function, which takes the hyperparameters as input and returns a scalar value representing the performance of the model. Optuna then minimizes the objective function by searching the hyperparameter space using the chosen optimization algorithm. Overall, the implementation of Optuna involves defining the search space for hyperparameters, defining the objective function, selecting the optimization algorithm, and visualizing the optimization process and results. This process can be automated using Optuna, saving time and resources compared to manual hyperparameter tuning.

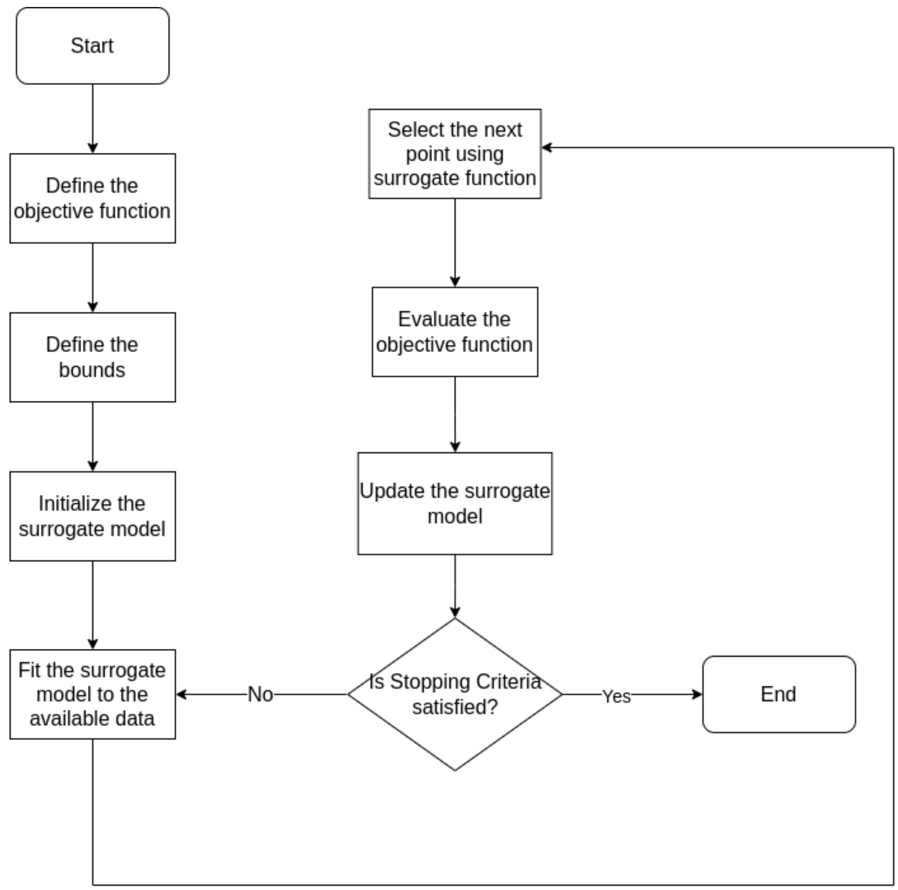


Figure 2. Flowchart depicting the functionality of BO

These optimization algorithms use the objective functions MAC, FIM and MAC & FIM to evaluate the fitness of each candidate solution and to narrow the search space to find improved solutions. These represent the quantity to be optimized, which in this study is the location of sensors for SHM.

Objective Functions:

Objective functions are employed in OSLP to assess the calibre of a specific sensor configuration. In OSLP, the MAC and the FIM are the two most frequently utilised objective functions. This study uses both with an additional criterion MAC & FIM that combines the advantages of both.

MAC AS OBJECTIVE FUNCTION

OSL can be determined by evaluating the non-diagonal elements of the MAC. A smaller value of the MAC's non-diagonal elements indicates better sensor placement. The size of these non-diagonal elements can be computed by maximizing their maximum value. Consequently, we can establish the following minimizing objective function:

$$F1(y_1, y_2, y_3, \ldots, y_s) = \max_{i \neq j} \mathrm{mac}_{ij} \tag{9}$$

The lower the value of the objective function F_1 (y_1, y_2, y_3, ..., y_s), the more suitable the sensor locations are.

FIM AS OBJECTIVE FUNCTION

By utilizing FIM as a criterion, we can guarantee the attainment of the most accurate estimation of the experimental modal parameters. Additionally, by placing sensors on high-amplitude vibration nodes, the measurement data's resistance to noise is improved. Therefore, we can establish a minimizing objective function by maximizing the Frobenius norm $||Q||_F$:

$$F1(y_1, y_2, y_3, \ldots, y_s) = -\,||Q|\,|_F \tag{10}$$

MAC & FIM AS OBJECTIVE FUNCTION

To ensure enhanced linear independence of MSV, improved noise resistance of measurement data, and optimal estimation of experimental modal parameters, a comprehensive approach is proposed in this study. The methodology incorporates the utilization of MAC and FIM in the following manner:

The initial step involves obtaining a row vector, denoted as $\Phi_{p\times p}$, through the primary column QR decomposition of the transpose of matrix Φ. This row vector is selected to maximize the norm, thereby identifying the OSL based on FIM. Subsequently, matrix Φ is partitioned into two sub-matrices: $\Phi_{p\times p}$ and $\Phi(q-p) \times p$.

By employing an optimization algorithm with an objective function $F_1(y_1, y_2, y_3, ...y_s)$, the study proceeds to derive $s-p$ row vectors contained in the $\Phi(q-p)\times m$ matrix, along with the p row vectors within the $\Phi_{p\times p}$ matrix. These s row vectors represent the sensor locations that collectively minimize the non-diagonal terms of their MAC. Consequently, the optimal sensor sites can be determined based on the MAC criterion.

RESULTS:

We took up an object to provide a demonstration of our idea, the cantilever beam. We rendered the cantilever beam on ANSYS software and performed modal analysis on it. The modal analysis resulted in locating 1000 points of deformation in the beam. We then performed different optimization techniques on the 1000 points to optimize it to 15 points. The X, Y, Z coordinates of those points are given in the Table 1 and the best result along with actual modeshapes are given from figure 3-9.

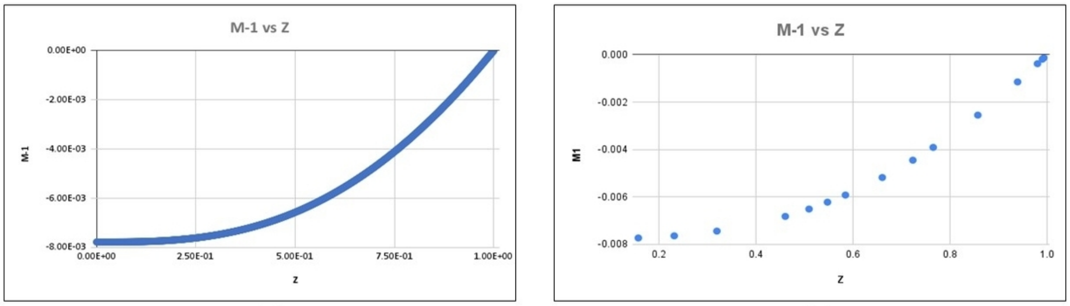
Note: OF is Objective Function, OA is Optimization Algorithm

CONCLUSION AND FUTURE WORK:

The locations of sensors and the number of sensors used for monitoring a structure directly affect how reliant the data and the monitoring process are. Continual use of numerous sensors through the life of a structure can significantly add to the cost of maintenance of the structure. By carefully selecting the locations of sensors, it is possible to capture the most vital information about the structure's behaviour and more accurately

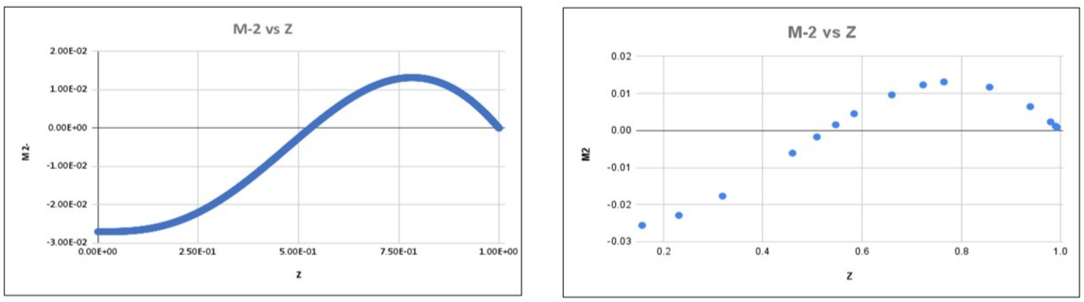
TABLE I. This table shows the OSL along with their respective MAC and FIM values upon using various optimization techniques and objective functions

OA	OF	OSL	MAC value	FIM value
PSO	MAC	322, 746, 485, 505, 337, 171, 842, 412, 132, 418, 376, 661, 995, 998, 925	0.119	7.463
PSO	FIM	999, 789, 400, 977, 652, 992, 527, 998, 854, 748, 734, 482, 975, 454	0.087	9.380
PSO	MAC and FIM	981, 994, 471, 925, 906, 298, 933, 674, 973, 373, 316, 954, 354, 861, 923	0.298	9.812
BO	MAC	821, 461, 494, 769, 635, 477, 945, 810, 886, 80, 661, 484, 993, 193, 295	0.115	7.838
BO	FIM	985, 890, 870, 986, 979, 298, 403, 962, 994, 203, 360, 892, 70, 836, 224	0.165	8.762
BO	MAC and FIM	247, 463, 340, 961, 216, 475, 259, 997, 500, 703, 453, 82, 879, 318, 853	0.141	8.267
Optuna	MAC	993, 394, 960, 298, 996, 916, 481, 357, 990, 983, 120, 546, 329, 231, 657	0.103	8.619
Optuna	FIM	247, 463, 340, 961, 216, 475, 259, 997, 500, 703, 453, 82, 879, 318, 853	0.141	8.267
Optuna	MAC and	489, 579, 883, 862, 229, 616, 774, 329, 970, 494, 291, 660, 997, 929, 755	0.122	7.378



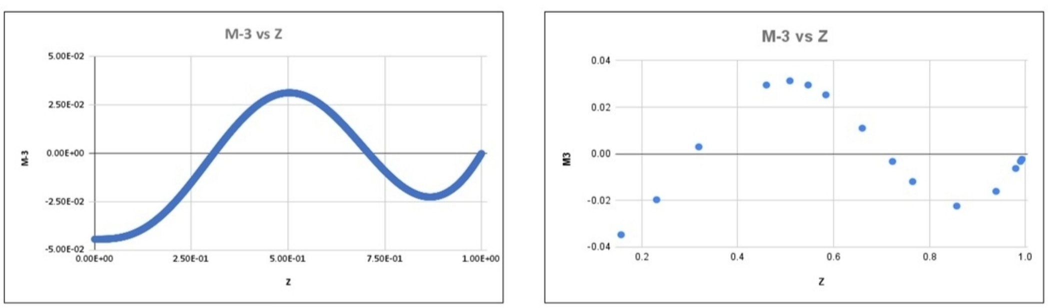
(a) (b)

Figure 3. Comparison of first mode shape results. (a) Modal analysis results. (b) Optimized results.



(a) (b)

Figure 4. Comparison of second mode shape results. (a) Modal analysis results. (b) Optimized results.

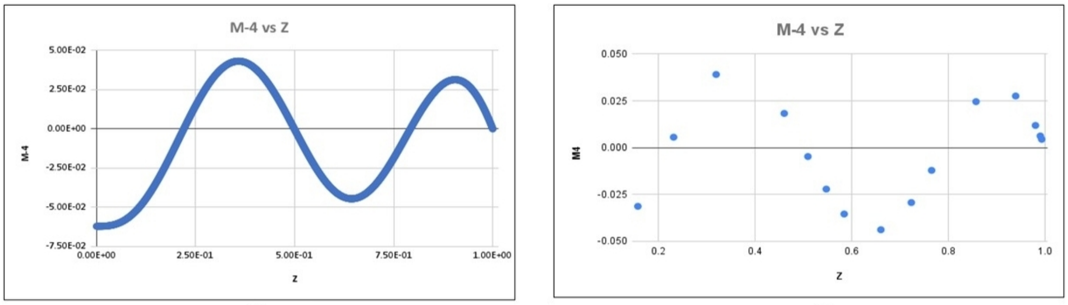


(a) (b)

Figure 5. Comparison of third mode shape results. (a) Modal analysis results. (b) Optimized results.

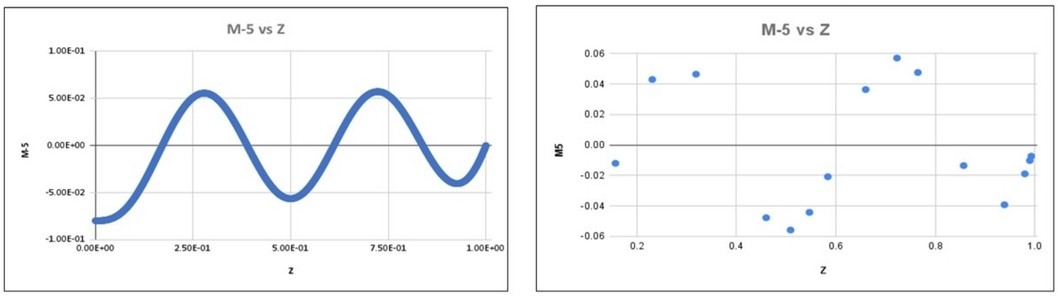
assess its condition while optimizing the sensors locations. This study aims to evaluate the suitability of sensor locations for modal analysis in a FEM model using the MAC and the FIM.

The study uses an ensemble model of multiple optimization techniques, such as PSO, BO, and Optuna, and uses MAC, FIM, and MAC & FIM as the objective functions for performing optimization on sensor locations on a Cantilever beam. In conclusion, enhancing the linear independence of mode shape MSV improves their resistance to noise in the measurement data, leading to heightened accuracy and reliability.



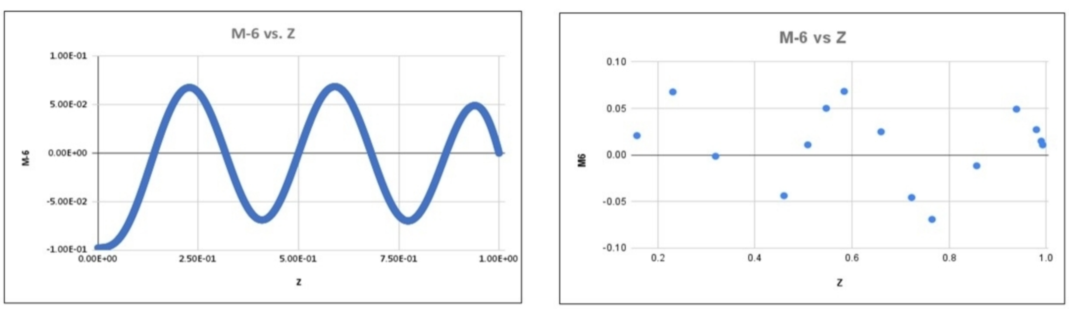
(a) (b)

Figure 6. Comparison of fourth mode shape results. (a) Modal analysis results. (b) Optimized results.



(a) (b)

Figure 7. Comparison of fifth mode shape results. (a) Modal analysis results. (b) Optimized results.



(a) (b)

Figure 8. Comparison of sixth mode shape results. (a) Modal analysis results. (b) Optimized results.

Upon implementation, we observed that: MAC & FIM as a criterion for OSLP can improve the accuracy of the MSV by enhancing their linear independence which improves the quality of the measurement data. By utilizing these criteria, the selection of OSL can lead to improved results in SHM.

Optuna required significantly fewer function evaluations than BO and PSO to find the optimal solution and it offers advanced features such as pruning and parallelization that further improve its efficiency and scalability.

In the future, this proposed ensemble optimization approach can be applied to structures of varying sizes, complexities, and uses. The model can be further extended to higher-dimensional search spaces and more complex structures to improve its efficiency and accuracy. Furthermore, the robustness of the OSL can be evaluated against changes in system parameters, such as boundary conditions and excitation force, to validate its practical application. These future works will help to enhance the model's applicability and utility in real-world applications.

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