Least-Square Support Vector Regression for the Prognosis of the Deteriorating Structure Under the Seismic Excitations Using Autoregressive Model

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ABSTRACT

In the current work, the prognostic behavior of the degradation in the space frame is carried out through the proposed data-driven framework based on autoregressive modelling and least square support vector regression. The acceleration responses were obtained at certain intervals after introducing the time dependent damage in the building. These responses are used to develop the damage index through filtering and statistical measures. The series of damage index is nonstationary in nature and therefore, the time varying autoregression (TVAR) modelling is carried out to obtain the change point. Next, for the prognosis, the algorithm performs surrogate modeling of observed degradation through least square support vector regression (LS-SVR) and the same is used to predict the trend of degradation. To increase accuracy and obtain the confidence bounds, a few mini datasets are created through omitting and shuffling the observation in the training dataset. The LSSVR predicts the degradation trend through each mini dataset. The mean and covariance of the prediction provides the best fit value along with upper and lower bounds. The advantage of SVR is that it can provide high-order approximations with sparse availability of samples. However, SVR optimized through sequential minimal optimization is time consuming and iterative in nature. Therefore, in the present study least square SVR is used for the model fitting and regression. Overall, the results highlight the potential advantage of creating mini datasets for taking advantages of LSSVR for the prognosis and obtaining the confidence bounds.

INTRODUCTION

Prognosis and health management (PHM) is the evolving tool for the reliable operations of machines and the proper functioning of the structural components in civil

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engineering systems. Further, it ensures timely fault detection and diagnosis, improved system reliability, effective maintenance, and prediction of remaining useful life. [1-6]. In the current work, prognosis is carried out through support vector regression (SVR) due to its versatile nature and ability to perform in sparse sample availability.

The support vector machine (SVM) is less prone to overfitting and has the advantage of providing high-order approximations with sparse availability of samples [7]. Louen, Ding and Kandler [8] use the SVM classifier to develop the health indicator and then predict the RUL using the Weibull function. Huang, Wang, Li, Zhang and Liu [9] reviewed the various algorithm based on SVM for the prognosis of the different systems. Yan, Wang, Wang, Chang and Muhammad [10] identify the degradation state of the bearings using the SVM classifier. Instead of using SVM as a classifier, several researchers use it as a regressor to predict the RUL [5]. Khelif, Chebel-Morello, Malinowski, Laajili, Fnaiech and Zerhouni [11] directly map the health indicator values using support vector regression (SVR). Xue, Zhang, Cheng and Ma [12] predict the RUL of lithium-ion batteries using Kalman filter and optimized SVR. Generally, SVM and SVR are optimized through sequential minimum optimization, which is iterative and time-consuming. Thus, least-square support vector regression (LS-SVR) is introduced as an alternative by Suykens and Vandewalle [13]. Qu and Zuo [14] optimize the hyperparameters of the LS-SVR through a genetic algorithm and provides an algorithm for the prognosis of the machine condition.

For the study, the building frame is selected and degraded under suitable time dependent deterioration law. It is excited under different seismic excitations to obtain the output acceleration responses, which were used to build a damage index. This damage index changes its trend due to certain seismic events, and time instant is known as the change point. The change point is identified through time-varying auto regression (TVAR) modelling. The prognosis of the damage index is carried out through least square support vector regression (LSSVR), which is applied to the mini datasets for obtaining the mean prediction curve and confidence intervals. The organization of the paper is as follows: The introduction discusses the literature review and explains the proposed work briefly, next section discusses the TVAR and LSSVR. Further, the algorithm is presented, followed by results and discussion. Finally, the conclusion is presented.

AUTOREGRESSION MODELLING

Autoregressive (AR) modelling is carried out for the stationary time series. However, the damage index pattern obtained in the present study is a non-stationary time series. Thus, time adaptive AR coefficients are needed to reconstruct the original pattern. Therefore, the time varying retrogression modelling is carried out for obtaining real time coefficients. The sudden change in the coefficient value will imply the change in the statistical pattern of the damage index. This sudden change signifies the occurrence of the change points. The details of the TVAR can be found in following literature [15].

LEAST SQUARE SUPPORT VECTOR REGRESSION

The support vector machine (SVM) is proposed by Vapnik [16] and the technique has two categories i.e., (i) classifier and (ii) regressor. SVM is the supervised learning algorithm trained by optimization theory that derives the weights and biases from statistical concepts and kernel functions. The advantage of the SVM is that the complexity and quality of the solution are independent of the dimension of the input dataset. Thus, it can provide the solution with the sparse availability of observations. The advantage of the SVM is that the complexity and quality of the solution are independent of the dimension of the input dataset. Thus, it can provide the solution with the sparse availability of observations. It maps the observations into high-dimensional feature space so that an optimal separating hyperplane can be constructed in that space.

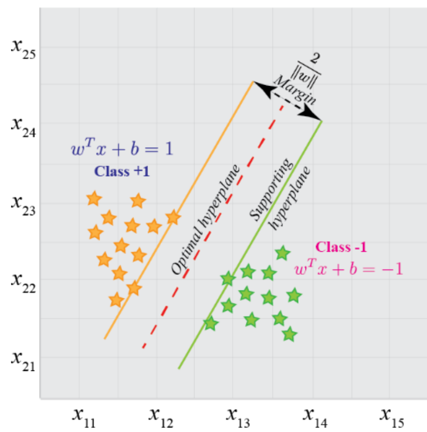

Figure 1. Illustration of the fundamental concept of support vector machines

Consider the training data set $X \in R$, is given by $X = \{(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)\}$, where, x is input pattern and y denotes the output pattern. Then, the optimal hyperplane separating the two different classes is given by $s(x) = \mathbf{w}^T x + b,$ here, $\mathbf{w} \in X,$ represents the coefficient vector of the hyperplane, and $b \in R$ is the bias. The concept of the support vector machine is represented in Figure 1. It can be shown that there should not be any point lies between the supporting hyperplane and the distance between them should be maximum, leading to the convex optimization problem as follows:

$$\begin{aligned} &\text{minimize} \quad && \frac{1}{2}\|w\|^2 \\ &s.t. && y_i(w^T x_i + b) - 1 \geq 0, \text{ where } y_i = \pm 1, \text{ and } i = 1, 2, \ldots, n \end{aligned} \tag{1}$$

The final solution of the above equation is given by:

$$s(x) = \sum_{i \in SV} a_i y_i x_i^T x' + b^* \tag{2}$$

where, SV represents the support vectors. The equation is modified for the nonlinear decision boundary using kernel function in which the samples are classified in the higher dimensions. The equation will be modified as

$$\begin{aligned} b^* &= y' - \sum_{i \in SV} \alpha_i y_i K(x_i, x') \\ s(x) &= \sum_{i \in SV} a_i y_i K(x_i, x') + b^* \end{aligned} \tag{3}$$

Where $K(x_i, x')$ represents the kernel function. The kernel function can be polynomial function $K(x,y) = ((x.y)+1)^d, d = 1,2,\ldots,n$ or the two-layer neural SVM function $K(x,y) = \tanh\left(\phi(xy)+\theta\right)$.

LS-SVM is another of SVM regression. Suykens and Vandewalle [13] replaces the inequality constraints through equality constraint by introducing errors. Thus, the least squares version of the SVM classifier is obtained by formulating the conventional problem as

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2}\|\mathbf{w}\| + \gamma \frac{1}{2}\sum_{i=1}^{n} e_i^2, \\ &s.t. \qquad y_i\left[w^T x_i + b\right] = 1 - e_i,\ \mathrm{i} = 1,\ldots,n. \end{aligned} \tag{4}$$

The corresponding Lagrange function in primal form is written as

$$L_p = \frac{1}{2}\|\mathbf{w}\| + \gamma \frac{1}{2}\sum_{i=1}^{n} e_i^2 - \sum_{i=1}^{n}\alpha_i \left\{ y_i\left[w^T x_i + b\right] - 1 + e_i \right\}, \tag{5}$$

where α_i are Lagrange multipliers. Minimizing the Lagrangian and the obtained condition can be written in the matrix form as follows:

$$\begin{bmatrix} I & 0 & 0 & -Z^T \\ 0 & 0 & 0 & -Y^T \\ 0 & 0 & \gamma I & -I \\ Z & Y & I & 0 \end{bmatrix} \begin{bmatrix} w \\ b \\ e \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \overline{1} \end{bmatrix} \tag{6}$$

where $Z = \left[x_1^T y_1; x_2^T y_2 \ldots; x_n^T y_n\right], Y = \left[y_1; \ldots; y_N\right], \overline{1} = [1;\ldots;1]$, $e = \left[e_1; e_2 \ldots; e_n\right]$, and $\alpha = \left[\alpha_1; \alpha_2 \ldots; \alpha_N\right]$. The solution can also rewrite as

$$\begin{bmatrix} 0 & -Y^T \\ Y & ZZ^T + \gamma^{-1} I \end{bmatrix} \begin{bmatrix} b \\ \hline \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ \hline \overline{1} \end{bmatrix}. \tag{7}$$

Using the Mercer's condition to the matrix $ZZ^T = \Omega_{ij} = y_i y_j x_i^T x_j = y_i y_j K(x_i, x_j)$, the above equation can be solved easily to obtain the solution of α ,and b .

ALGORITHM

The proposed framework takes the acceleration responses in the data acquisition step. The acceleration responses are processed to develop the damage index using singular spectral analysis and statistical distances. Since the LS-SVR and grid search optimization works on the training dataset, it gives improper results if the dataset uses observations from the beginning even after the change point occurs. Therefore, the dataset needs to be revised when the change point occurs. The detection of change points is carried out using the TVAR coefficients. The sudden change at any time instant in the TVAR coefficients will be considered as change point. If a change point occurs, the training dataset is re-evaluated. Once the training dataset is finalized, the mini datasets are created through random shuffling and omitting the observations. For each mini dataset, the prognosis is carried out using LSSVR. The hyperparameters are optimized through grid search optimization. Thus, several prognosis curves are obtained through each mini-dataset. These curves are used to calculate the mean prediction curve and the confidence interval bounds. The steps of the algorithm are presented in Figure 2

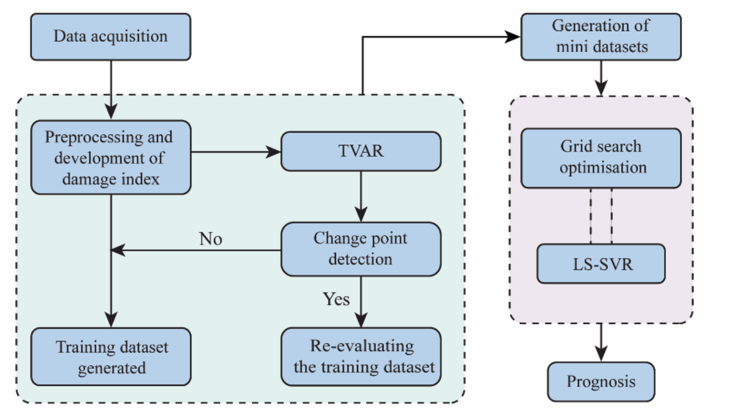

Figure 2 Illustration of the working mechanism for prognosis

RESULT AND DISCUSSION

The damage index in the current study is developed using the combination of singular spectral analysis along with recursive autoregressive modelling. Finally, the statistical measures called Bhattacharyya distance between auto regressive coefficient clusters is used as damage index. The details can be found in the following literature [17]. The building frame is selected and degraded under suitable time dependent deterioration law. It is excited under different seismic excitations to obtain the output acceleration responses which were used to build a damage index. Figure 3 (a) represents the change in the damage index. Further, for evaluating the training dataset, the perturbations are added to the damage index and it has been reconstructed through TVAR. The sudden change in the time varying AR coefficient will identify the occurrence of change point. Figure 3 (b) represents the change point detection using time varying AR coefficients.

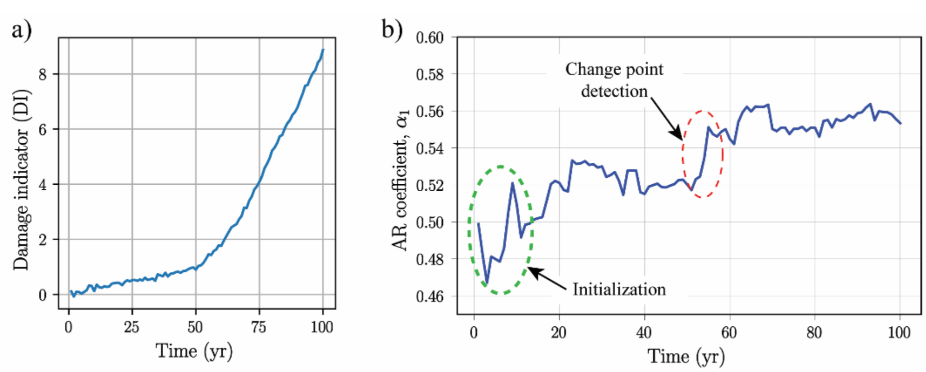

Figure 3 (a) Damage index, and (b) time varying AR coefficient for change point detection

For the application of LSSVR, it is important to decide the training dataset. Until the change point occurs, the observations from the beginning till the time of prediction are considered in the training dataset. After the change point occurs, the observation from the change point till the time of prediction is considered in the training dataset. For each of the training datasets, several mini datasets are created through shuffling and omitting the observations. The prediction curve is calculated for each mini dataset and then the mean and confidence bounds are obtained. The advantage of creating the mini dataset is that variation in training data is considered resulting in better prediction and obtaining the confidence interval for such a deterministic approach. Figure 4 (a), (c), and (d) are the prognosis results obtained through LSSVR directly using the training dataset. Figure 4 (b), (d) and (f) are the results obtained considering the mini datasets. Figure 4 (e) and (f) provide the prediction results without re-evaluating the training dataset, still the proposed algorithm works better.

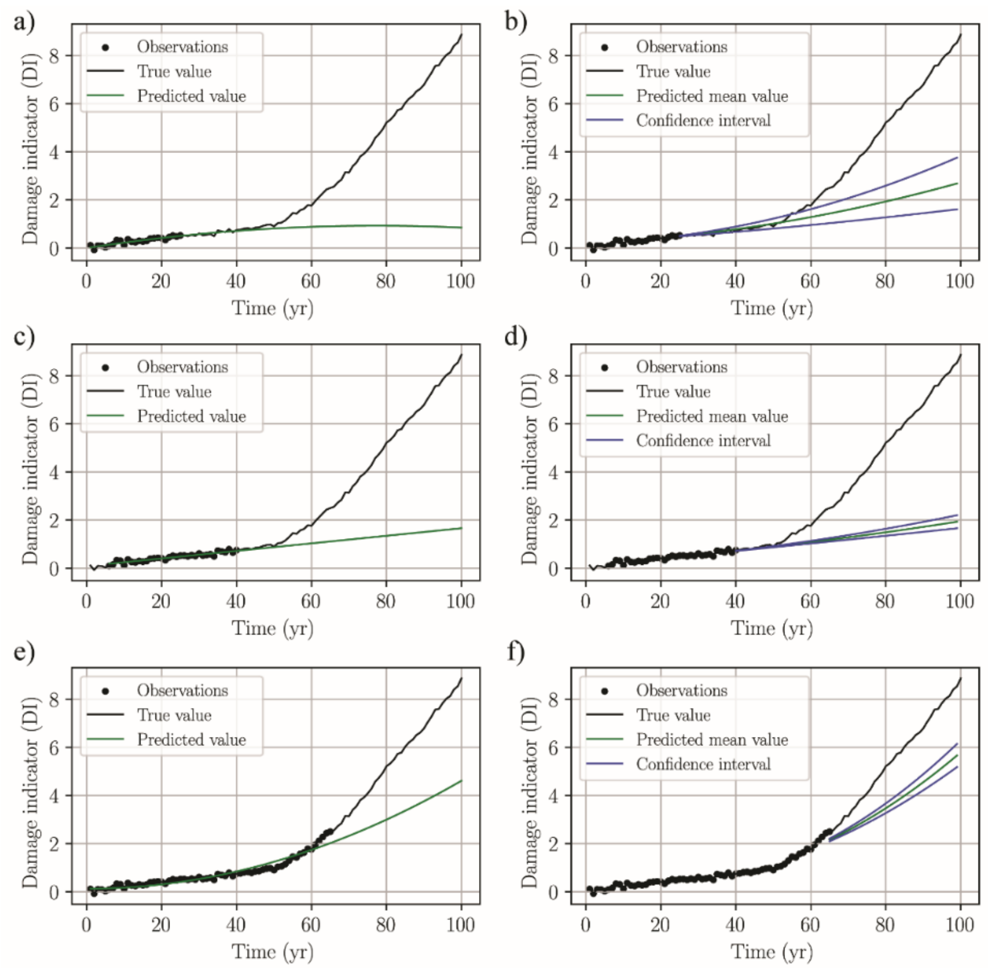

Figure 4. Prognosis of the degradation index without considering change point, (a), (c), (e) regular approach, and (b), (d), (f) proposed approach using mini datasets.

Further, the comparison between the prediction results of the proposed framework with the conventional approach is carried out using mean square error (MSE) and coefficient of determination (COD).

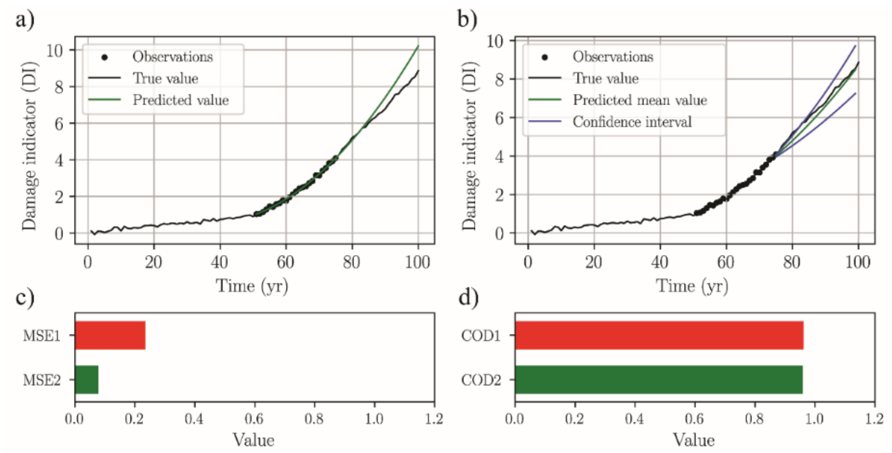

Figure 5 Prognosis of damage index at 65 years considering change point (a) regular approach, (b) proposed approach, (c) MSE, and (d) COD.

Figure 5 (a), and (b) shows that the proposed framework predicts the degradation trend more accurately with mean square error less than 0.1. The notation '1' and '2' in figure below represents the conventional and proposed approach simultaneously.

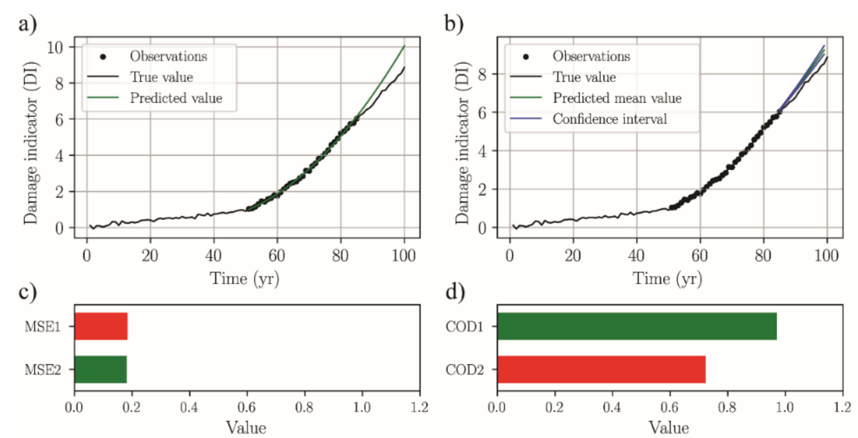

Figure 6 Prognosis of damage index at 85 years considering change point (a) regular approach, (b) proposed approach, (c) MSE, and (d) COD.

Further, a similar inference can be made for Figure 6. It can be clearly observed that as the size of dataset grows, the predicted mean of proposed approach tends to coincide with the conventional prediction. However, the confidence interval holds an added advantage.

CONCLUSION

The current work introduces TVAR for change point detection and concept of the mini datasets to make results slight probabilistic. This proposed approach provides confidence interval in the prediction results, using LS-SVR, which is deterministic in nature. The LS SVR is optimized through grid search optimization. The efficacy of the proposed algorithm is examined through the regular prediction approach. The results of the prognosis confirm the accuracy and effectiveness of the proposed mini datasets concept in the LS-SVR framework for predicting degradation.

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