Evolutionary Sensor Network Design for Structural Health Monitoring of Structures with Time-Evolving Damage

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ABSTRACT

Structural health monitoring (SHM) aims to assess damage intensity and provide engineers with data to make informed maintenance and repair decisions. SHM systems collect crucial information for evaluating a structure's current state, enabling appropriate maintenance decisions and loss mitigation. Therefore, it is crucial to acquire damage-sensitive data by using a well-designed SHM system that is optimal in terms of expenses as well as functionality. In this research, we present an optimal sensor placement framework that considers two stages of the structure's lifespan: (1) an *early-stage* pre-posterior design, and (2) a periodically updated sensor design in the *operational stage*.

When the sensors are designed initially in the pre-posterior stage, there is no data available to make informed design assumptions for designing the SHM system. As a result, all the uncertainties and damage evolution models for the structure need to be modeled probabilistically based on reasonable assumptions derived from historical perspective and engineering judgment. The early-stage design of an SHM system initiates the data acquisition and serves two primary purposes: (1) helps update the current state of the structure, and (2) supports data-informed maintenance decisions. As the structure degrades over time, despite periodic maintenance, it is bound to approach the limiting or critical damage state. This warrants an even better inference of damage state with the goal of avoiding the worst scenario of failure. In addition, another reason to update the sensor design while in the operational stage is to optimize the SHM system by making it more suitable to the current structural state and benefit from the data acquired through the pre-posterior design. Periodically updating the design yields the best risk to reward for an SHM system in terms of its expenses and functionality. We demonstrate the application of the proposed framework on a real-world complex miter gate structure.

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INTRODUCTION

The objective of SHM is to evaluate the severity of damage and furnish engineers with relevant information for making informed choices about maintenance and repairs. An SHM system achieves this target by gathering critical data that helps to determine the present condition of the structure (diagnostics) and probabilistically forecasting the evolution of the damage state (prognostics). This not only enables engineers to make appropriate maintenance decisions for the current diagnosed state but also to plan future actions, ensuring the reliability and safety of the structure.

While the data obtained from an SHM system facilitates informed decision-making and enhances our comprehension of the system, it is important to consider the costs involved in designing, developing, installing, maintaining, and operating the system. As such, the true potential of the SHM process can be unleashed by optimally designing the SHM system that aims at maximizing the Value of Information (see [1,2,3]) and would also be a crucial part of creating a digital twin for the lifecycle management of the structure (see [4,5]).

In this paper, an extension to our previous efforts (see [6-7]), we focus on the optimization effort that primarily targets the optimal "when" and "where" of data acquisition that fuels the SHM engine. We propose doing this by making two observations: (1) It is often observed that an SHM system is not necessary during the early pristine phase a structure's life cycle, when the risk of failure or catastrophic damage is minimal. Therefore, during this phase, the SHM system can be a cost liability rather than an asset. However, as the structure evolves, acquiring additional information about the structural state becomes increasingly valuable for decision-making. As the risk of failure increases, the need for more information about the structural state also increases. Consequently, the quantity of information gathered is directly related to the risk of an unwanted event occurring. Exploiting this relationship, we propose an approach that determines when additional resources should be allocated to gather more information, thereby optimizing the monitoring of the structure in the time domain; (2) The spatial arrangement of sensors depends on the damage that needs to be inferred. The proposed approach also obtains and updates the spatial arrangement of sensors that maximizes the target objective function. As a result, our proposed optimization framework covers both temporal and spatial dimensions.

In the following writeup, we motivate and demonstrate the proposed time-dependent sensor optimization framework on a case study involving structural health monitoring of a miter gate that is part of a lock system enabling navigation of inland waterways. The inland waterways navigation infrastructure is operated and maintained by the United States Army Corps of Engineers (USACE).

THE STRUCTURE, LOADING, AND DAMAGE EVOLUTION MODEL

Consider the Greenup miter gate shown in Fig. 1. Over its life cycle, the gate is subjected to uncertain loading, denoted by the random vector $H(t)$ with a realization $h(t) \in \Omega_{H(t)}$. The upstream and downstream water head is denoted by $h_{up}(t)$ and $h_{down}(t)$, such that $h(t) = \{h_{up}(t), h_{down}(t)\}$. The state of the miter gate is assumed to be completely defined by the loss of boundary contact (or a "gap") between the gate and the concrete wall at the bottom of the gate. This is referred to as a gap-length (a scalar state-parameter defined at time t), denoted by $\theta(t) \in \Omega_{\Theta(t)}$. At $\theta = \theta_{min} = 0$ inches (lower bound) the gate is pristine. Based on the suggestions from USACE field engineers, the limit $\theta = \theta_{max} = 180$ inches is assigned as the upper bound of loss of boundary contact at which point the gate is considered to be failed and nonoperational.

Figure 1. Greenup miter gate (figure adopted from Chadha et al. [1])

To capture the prior understanding of how the damage might evolve probabilistically, we utilize a piecewise multi-stage prior gap degradation model, which is explained in Section 5.1 of Chadha et al. [1], illustrated in Fig. 2. The figure also illustrates the mean gap curve and the standard deviation of gap over time, denoted by $\mu_{\theta(t)}$ and $\sigma_{\theta(t)}$ respectively. Another curve that is crucial to our proposed dynamic sensor design framework is the Coefficient of Variation (COV) of prior degradation model denoted by $\rho_{prior}(t) = \sigma_{\theta(t)}/\mu_{\theta(t)}$ (plotted in red with scale on the right y-axis in Fig. 2).

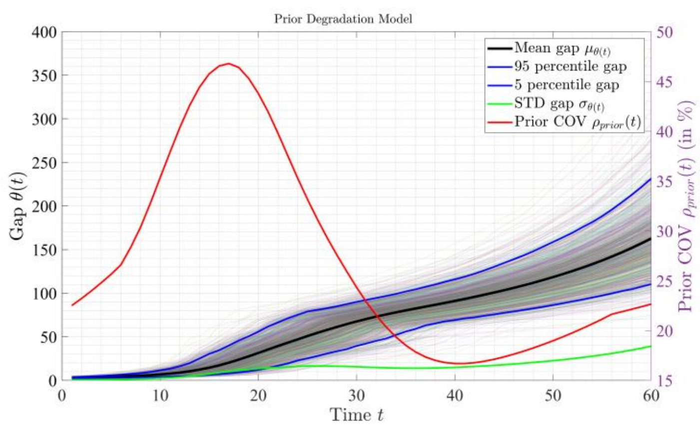

Figure 2. Probabilistic prior gap degradation model

Since sensor optimization over the lifecycle warrants sensor measurements for different loading conditions at different instances of time and damage level, it is necessary to simulate the ground truth. We do so by using a high-fidelity finite element model or its digital surrogate (see [8]). Let the observed measurements for a set of sensors at time t be denoted by $x(t) \in \Omega_{X(t)}$. We obtain $x(t)$ by adding the noise vector $\varepsilon(t) \in \Omega_{\zeta(t)}$ to the ground-truth value of sensor measurement obtained using the finite element method (FEM) model $g(\theta(t), h(t); t)$. That is,

$$x(t) = g(\theta(t), h(t); t) + \varepsilon(t) \tag{1}$$

Bayesian inference can then be used to infer the posterior distribution of the gap value (see Chadha et al. [1] and Ramancha at al. [9]) denoted by $f_{\Theta(t)|X(t)}\big(\theta(t)\big|x(t)\big)$.

THE OPTIMIZATION FRAMEWORK

The proposed sensor optimization framework spans *temporal* as well as *spatial* dimensions. We first consider the *temporal* dimension. In the time domain, the proposed sensor optimization occurs in stages. During the early stages of the structure's lifespan, there is no urgent need for additional information about its health. As a result, the uncertainty in the damage parameter is acceptable up to a certain point in time without any significant risk of unwanted events. We use the prior COV $\rho_{prior}(t)$ to quantify the uncertainty in the gap value since it's a measure of the standard deviation of gap relative to its mean. As time progresses, there comes a point where additional information is necessary to gain a better understanding of the structural health or to infer the posterior of the gap. We refer to this as the *stage 1* design that occurs at time $\bar{t}_1$.

To obtain the time $\bar{t}_1$ and the subsequent time for next stages, we perform a pre-posterior optimization of a Bayes risk function $\Psi_{\text{temporal}}(\rho(t);\ t)$ defined to capture the consequence of various levels of uncertainties $\rho(t) \in [0,1]$. We delineate sequential steps of reasoning to formulate $\Psi_{\text{temporal}}(\rho(t);\ t)$ as follows:

Step 1: For each time instance t, we consider the realizations of the prior distribution of the gap value, denoted by $f_{\Theta(t)}\big(\theta(t)\big)$, to be the possible true value. Then we consider various levels of uncertainty quantified by coefficient of variation $\rho(t) = \frac{\sigma(t)}{\theta(t)} \in [0,1]$. For each realization pair $\big(\theta(t), \rho(t)\big)$ we assume a normal distribution $N\big(\theta(t), \sigma(t) = \rho(t)\theta(t)\big)$ to quantify distribution of the gap value with $\theta(t)$ as its mean and coefficient of variation $\rho(t)$. If the realization $\theta(t)$ is less than θ_{max}, it implies that the assumed true gap at time t is *safe* (an event denoted by S), and if $\theta(t)$ is greater than θ_{max}, it denotes that the assumed true gap at time t is *not safe* (an event denoted by F). That is,

$$\begin{aligned} \theta(t) < \theta_{max} &\implies P(S) = 1 \text{ and } P(F) = 0 \\ \theta(t) > \theta_{max} &\implies P(S) = 0 \text{ and } P(F) = 1 \end{aligned} \tag{2}$$

Step 2: Let $\phi(t)$ denote a realization of $N\big(\theta(t), \sigma(t)\big)$. If $\phi(t)$ is greater than θ_{max}, it then reflects the case of nonoperational limit state, represented by the event $\hat{F}$. On the other hand, if $\phi(t)$ is less than θ_{max}, it then indicates the case of a safe structure, represented by the event $\hat{S}$.

Step 3: We now define the following conditional probabilities:

$$\begin{aligned} P\big(\hat{S}\big|F\big) = P\big(\hat{S}\big|S\big) &= \Phi\left(\frac{\theta_{max} - \theta(t)}{\rho(t)\theta(t)}\right) \\ P\big(\hat{F}\big|F\big) = P\big(\hat{F}\big|S\big) &= 1 - \Phi\left(\frac{\theta_{max} - \theta(t)}{\rho(t)\theta(t)}\right) \end{aligned} \tag{3}$$

Step 5: For each realization $\theta(t)$, and the coefficient of variation $\rho(t) \in [0,1]$ at time t, we define the expected risk/consequence $C(\theta(t), \rho(t); t)$ as:

$$C(\theta(t), \rho(t); t) = c_{10} P(\hat{F}|S).P(S) + c_{00} P(\hat{S}|S).P(S) + c_{11} P(\hat{F}|F).P(F) + c_{01} P(\hat{S}|F).P(F) \tag{4}$$

Here, c_{ij} is the nominal consequence cost of a decision by assuming that the structure is in state $i \in \{0,1\}$ while its true state is $j \in \{0,1\}$. Here, 0 denotes *safe*, while 1 denotes *not safe* (binary decision). Since these are nominal costs, we assume $c_{01} = 1$ (it's the worst case) and obtain other costs in terms of c_{01}: $c_{10} = 0.25\, c_{01}$; $c_{00} = 0\, c_{01}$; $c_{11} = 0.5\, c_{01}$.

Step 6: The temporal Bayes risk, denoted by $\Psi_{\text{temporal}}(\rho(t);\, t)$, as a function of the coefficient of variation $\rho(t) \in [0,1]$ is obtained as:

$$\Psi_{\text{temporal}}(\rho(t);\, t) = \int_{\Omega_{\Theta(t)}} f_{\Theta(t)}(\vartheta) C(\vartheta, \rho(t); t)\, d\vartheta \tag{5}$$

Finally, the optimal coefficient of variation at time t is obtained as:

$$\rho_{optimal}(t) = \underset{\rho(t)}{\operatorname{argmax}}\, \Psi_{\text{temporal}}(\rho(t);\, t) \tag{6}$$

At time $\bar{t}_1$, $\rho_{optimal}(t) < \rho_{prior}(t)$ and that is when additional information is required. The Fig. 3 illustrates the optimal coefficient of variation $\rho_{optimal}(\bar{t}_1) = 44\%$ for stage 1 design to be installed at time $\bar{t}_1 = 14$ months. An interpretation of this is that we don't need additional information about the structural state for up to 14 months since the risk of catastrophic incidence is minimal.

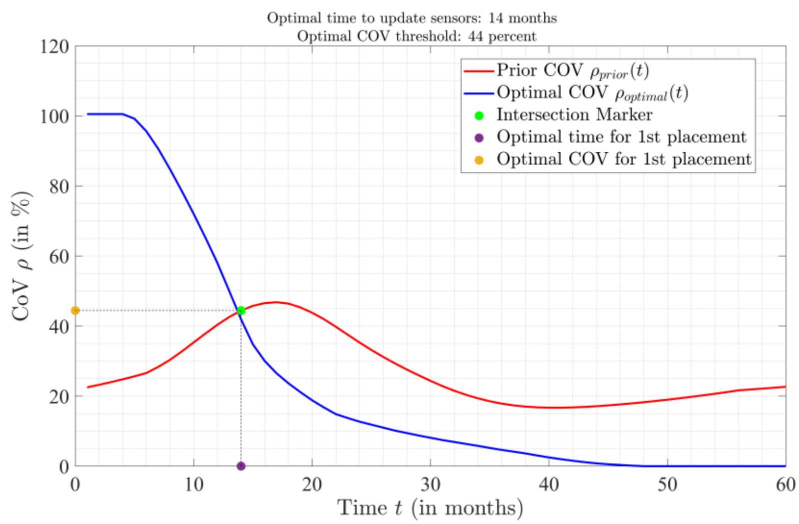

Figure 3. Prior vs. optimal coefficient of variation plot

Once we know the time $\bar{t}_1$, we obtain the spatial placement of the first sensor design consisting of 1 sensor that provides just the right amount of additional information.

Let $e[\bar{t}_i]$ denote an instance of the $i-$th stage sensor design consisting of $N_{sg}(e[\bar{t}_i])$ strain gauges with measurements $x_{e[\bar{t}_i]} \in \Omega_{X_{e[\bar{t}_i]}(\bar{t}_i)}$ such that $t \in [\bar{t}_{i-1}, \bar{t}_i)$. Let $\varepsilon_{e[\bar{t}_i]} \in \Omega_{\zeta_{e[\bar{t}_i]}(\bar{t}_i)}$ denote the noise vector. Using the acquired sensor data, the damage parameter can be probabilistically updated via Bayesian inference to obtain the posterior distribution of gap conditioned on acquired data $x_{e[\bar{t}_i]}(\bar{t}_i)$, denoted by $f_{\Theta(\bar{t}_i)|X_{e[\bar{t}_i]}(\bar{t}_i)}\left(\theta(\bar{t}_i)\middle|x_{e[\bar{t}_i]}(\bar{t}_i)\right)$. The optimal sensor design for the stage identified with time $\bar{t}_i$ is obtained as:

$$e^*[\bar{t}_i] = \underset{e[\bar{t}_i]}{\operatorname{argmax}} \Psi_{\text{spatial}}(e[\bar{t}_i];\ \bar{t}_i)\text{, where} \tag{7a}$$

$$\Psi_{\text{spatial}}(e[\bar{t}_i];\ \bar{t}_i) = E_{\Theta(\bar{t}_i)H(\bar{t}_i)\zeta_{e[\bar{t}_i]}(\bar{t}_i)}[\mathcal{L}(e[\bar{t}_i];\bar{t}_i)] \tag{7b}$$

Here, $\Psi_{\text{spatial}}(e[\bar{t}_i];\ \bar{t}_i)$ denotes the spatial Bayes risk which is defined as the expected value of the target risk $\mathcal{L}(e[\bar{t}_i];\bar{t}_i)$. For this paper, we use the target risk as the KL divergence defined as:

$$\mathcal{L}(e[\bar{t}_i];\bar{t}_i) = KL\left(f_{\Theta(\bar{t}_i)|X_{e[\bar{t}_i]}(\bar{t}_i)}\left(\theta(\bar{t}_i)\middle|x_{e[\bar{t}_i]}(\bar{t}_i)\right)\middle|\middle|f_{\Theta(\bar{t}_i)}(\theta(\bar{t}_i))\right) \tag{8}$$

The design $e^*[\bar{t}_1]$ obtained by using Bayesian optimization on Eq. (7a) is then used to obtain the updated posterior gap degradation model $f_{\Theta(t)|X_{e[\bar{t}_1]}(t)}\left(\theta(t)\middle|x_{e[\bar{t}_1]}(t)\right)$ valid for $t > \bar{t}_1$ (see the details of Bayesian optimization algorithm in Yang et al. [3]). The updated posterior gap degradation model can then be used to obtain the next stage $\bar{t}_2$, which then can lead to the second stage optimal design $e^*[\bar{t}_2]$. For the simulated example, $\bar{t}_2 = 17$ months with $\rho_{optimal}(\bar{t}_2) = 34\%$. This process continues till an acceptable level of coefficient of variation is achieved and beyond which, the optimization effort ceases to yield value. Figure 4 illustrates the designs $e^*[\bar{t}_1]$ and $e^*[\bar{t}_2]$.

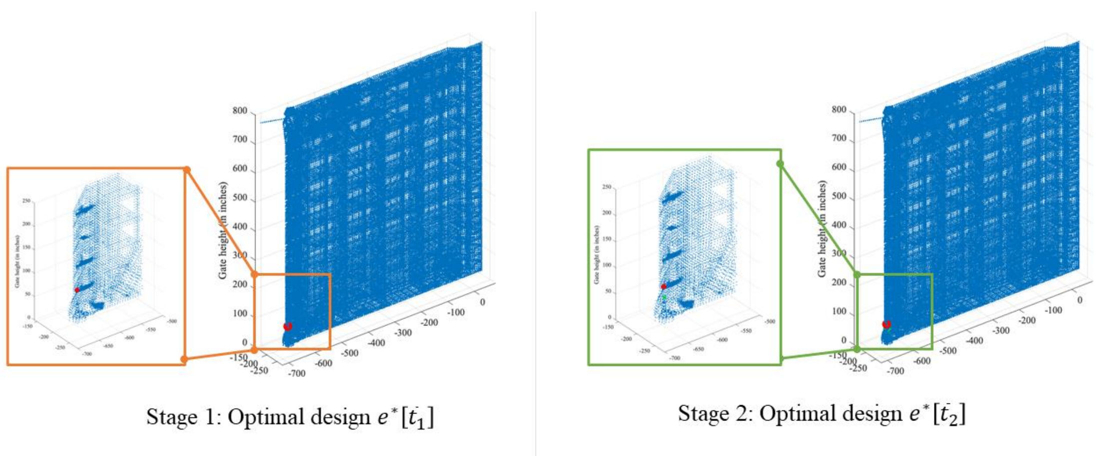

Figure 4. Optimal stage 1 (left) and stage 2 (right) sensor designs with 1 and 2 sensors respectively.

CONCLUSIONS

This article concisely details the mathematical formulation for a dynamic sensor optimization framework that spans both the temporal as well as spatial dimensions. By considering the uncertainty in the damage parameter over time, we use pre-posterior information analysis to identify the optimal time instance for updating the sensor design, based on the need for additional information. We then install an additional sensor that yields an acceptable optimal coefficient of variation. The spatial arrangement of the sensors is designed to maximize the gain in information relative to the prior knowledge at that instance of time. By dynamically maximizing the Value of Information (defined as needed), our proposed optimization framework enables an SHM system with a higher risk-to-reward ratio.

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