Implementation of Decision Analysis on a Structural Health Monitoring System Applied to a Bridge Benchmark Study

GABRIEL A. DEL POZO, BJØRN T. SVENDSEN
and OLE ØISETH

ABSTRACT

Structural health monitoring (SHM) of bridges addresses the need for efficient and cost-effective monitoring of structures. The purpose of SHM is to detect structural damage and provide information related to maintenance, inspection, and repair. This information would then be supplied to asset managers in order to make improved maintenance decisions. This paper presents an investigation into the connection between SHM and decision-making via Bayesian decision theory and the value of information (VoI) obtained from SHM. The results and properties of an SHM damage classifier are used to update the probabilities of a tree-based decision model. Two analyses are included. Firstly, a sensitivity analysis of the value of information (VoI) of an SHM system for varying cost ratios is performed. The SHM system was applied to a benchmark study in which the dynamic behaviour of a steel bridge was measured for both undamaged and damaged structural conditions. Secondly, the relationship between the expected costs of the available actions and the size of the sensor network of the SHM system is obtained by selecting sub-sets from the complete measurement system utilized in the benchmark system. The applicability of SHM results for informing and updating damage probabilities is demonstrated.

INTRODUCTION

Structures close to reaching their design service life, such as a significant proportion of European bridges [1], require careful and continuous control and supervision to ensure safe usage. Structural health monitoring (SHM) has, in the last two decades, emerged as an alternative or complement to traditional control practices. The overall objective is to provide the authorities with the correct information such that infrastructure management is optimal.

Decision theory provides infrastructure management the possibility to analyse and obtain expected utilities of possible actions. Decision theory is based on risk analysis of a number of outcome costs or utilities where the probability of a state of nature combined with an action would result in a given outcome. The state of the art of SHM

Department of Structural Engineering, Norwegian University of Science and Technology, 7491 Trondheim, Norway

systems indicates the feasibility of detecting damage on real structures and supplying error rates of these SHM damage classifiers [2][3]. This allows the usage of SHM systems as input for decision-making [4].

It remains important to verify that the costs associated with implementing an SHM system are compensated by the benefit of supplying additional information for decision-makers. This analysis is approached using the concept of the value of information (VoI) of SHM, which is, in simple terms, the difference in expected utilities between implementing an SHM system or not for a given structure. In other words, the challenge is to consolidate the relationship between damage detection and utility obtained from better decisions. The increased interest in SHM in the last couple of decades also implies heightened research interest in the VoI of SHM [5].

This paper explores the effects that different variables of an SHM system have on the expected costs of maintaining a bridge. This is achieved by defining a decision model that considers the actions of doing nothing, repairing, or inspecting the bridge. The model also evaluates the implementation of the SHM system on the structure by employing properties and results from the SHM damage classifier to update the probabilities of failure of the structure. The classifier indicates positive or negative indications of damage and has false positive and false negative rates of error associated with these indications. The error rates of the damage classifier are obtained from implementing it on a benchmark study on the Hell Bridge Test Arena (HBTA), where the responses of the bridge to dynamic excitation in both undamaged and damaged structural conditions were captured with a dense sensor network [6].

The results present a sensitivity analysis of the VoI of the SHM system implemented at HBTA for different repair/SHM and failure/SHM cost ratios. Additionally, the relationship between the size of the sensor network and the expected utilities of the available actions is obtained.

EXPERIMENTAL STUDY

The HBTA, shown in Figure 1, is an open-deck steel railway truss bridge. The bridge consists of two vertical walls, a deck, and lateral bracing located below the deck. After more than 100 years of operation, the bridge was moved to a test facility located in Norway. The length and width of the bridge are 35 m and 4.5 m, respectively. The bridge serves as a full-scale experimental test bridge for damage detection and SHM [7].

Figure 1. The Hell Bridge Test Arena

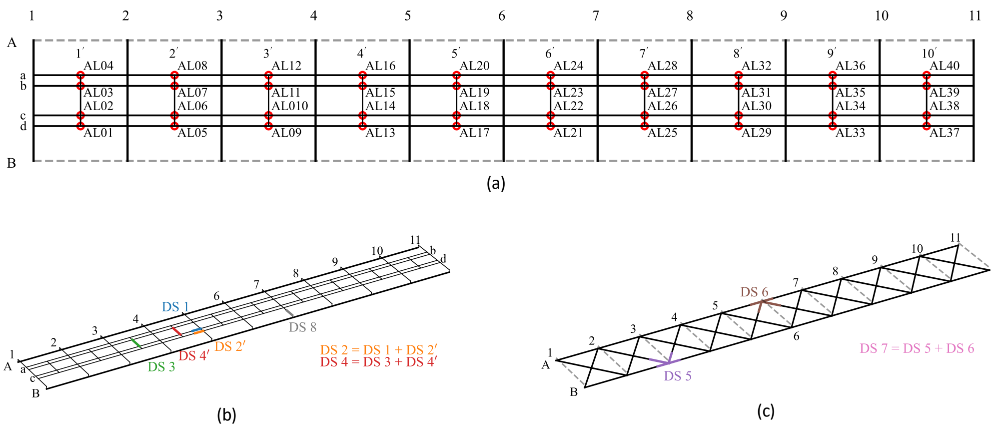


Figure 2. Overview of the experimental conditions. (a) Location and numbering of accelerometers on the bridge deck. (b) Damage locations on bridge deck. (c) Damage locations on the lateral bracing

The structural system of the bridge, shown in Figure *2*a, is composed of stringers a - d, with cross beams between them labelled 1' - 10'. The sleepers are supported by floor beams, identified with labels 1 - 11. These floor beams are connected to the vertical walls labelled A and B. The lateral bracing of the structure is connected to the lower chords of these walls.

An extensive experimental campaign was carried out on the HBTA during autumn 2021. The structure was excited by a modal vibration shaker in both damaged and undamaged conditions. The modal vibration shaker was configured to provide white noise as dynamic loading, while the bridge response was captured using 40 single-axis accelerometers (Dytran 3055D3) located below the bridge deck. The accelerometers were distributed along and across the bridge deck, as shown in Figure *2*a.

Table I describes the different damaged and undamaged conditions in which the structure was excited while recording its response. Figure *2*b and Figure 2c indicate the location of the eight damage states. These damage states were imposed on different connections and were introduced by temporarily removing all the bolts of these joints. Four different damage types were selected: stringer-to-floor beam connections (DS 1 and DS 2), stringer cross beams (DS 3 and DS 4), lateral bracing connections (DS 5 – DS 7), and connections between floor beams and main load-carrying members (DS 8). The two undamaged states, considered the baseline, correspond to the undamaged condition of the structure, i.e., before any damage was imposed and after repairing all the damages. 80 sets of acceleration time series were obtained for each state condition adding up to 800 tests. Additional documentation of the data acquisition system, signal processing, and experimental conditions are available in Svendsen et.al. [7].

TABLE I. OVERVIEW OF THE STATES OF THE STRUCTURE DURING THE EXPERIMENT

Label	State condition	Type	Description
UDS 1	Undamaged	Baseline condition	Before all damage state conditions
UDS 2	Undamaged	Baseline condition	After all damage state conditions
DS 1	Damaged	Stringer-to-floor beam connection	Single connection damaged
DS 2	Damaged	Stringer-to-floor beam connection	Multiple connections damaged
DS 3	Damaged	Stringer cross beam	Main part of single cross beam removed
DS 4	Damaged	Stringer cross beam	Main parts of multiple cross beams removed
DS 5	Damaged	Lateral bracing connection	Single connection damaged
DS 6	Damaged	Lateral bracing connection	Single connection damaged
DS 7	Damaged	Lateral bracing connection	Multiple connections damaged
DS 8	Damaged	Connection between floor beam and main load-carrying member	Single connection damaged

DAMAGE-SENSITIVE FEATURES AND CLASSIFIER

AR parameters are selected to characterize the acceleration time series as damage-sensitive features, as done in previous studies [8][9]. An AR model utilizes past data points of a time series to obtain the present data point. The quantity of past data points used by the model is the order of the model. The p-order autoregressive model, AR(p), for a given time series is defined as [10]:

$$y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \varepsilon_t \tag{1}$$

where y_t and y_{t-j} are the data points at indexes t and t - j, φ_j is the autoregressive parameter j, and ε_t corresponds to the residual of the model at index t.

An AR(5) model is selected and each accelerometer time series is characterized by five coefficients. This leads to 200 features per test. The complete set of AR coefficients become a matrix with 800 rows and 200 columns, where each row corresponds to a specific structural state and each column to an AR coefficient for a specific sensor. This matrix is divided for training and testing, and the Mahalanobis square distance (MSD) algorithm is applied to obtain damage indices (DIs). These DIs are the numerical values of the MSD, which indicate the distance between point z_i and the mean $\bar{x}$ of a sample distribution, given by:

$$DI_i = (z_i - \overline{x}) C^{-1} (z_i - \overline{x})^T \tag{2}$$

where C is the covariance matrix. C and $\bar{x}$ are obtained from the training matrix.

The approach to defining the threshold, which separates damaged behaviour from baseline behaviour for 99 % confidence, is based on Monte Carlo simulations [11].

The false positive rate (FPR) and false negative rate (FNR) of the SHM damage classifier are obtained based on the fact that the true state of the structure, undamaged or damaged states, are known. The FPR is obtained by dividing the number of times the classifier indicates damage from the tests when the real state is undamaged by the total quantity of undamaged tests. Conversely, the FNR is obtained by dividing the number of times the classifier indicates no damage from the tests when the real state is damaged by the total quantity of damaged tests.

DECISION ANALYSIS FOR STRUCTURAL HEALTH MONITORING

Bayesian decision theory involves incorporating Bayesian probability updating into the decision trees commonly used in decision theory. Decision trees are models where each combination of a set of available actions $a_i \in A$ with possible states of nature $\theta_i \in \Theta$, which are associated with probabilities, are assigned a value or cost $u_i \in U$. An experiment increases the amount of available information for the decision model and its outcomes can provide additional certainty regarding the probabilities of states of nature. Additional certainty is implemented in a-posteriori and pre-posteriori decision trees by adding layers of experiments $e_i \in E$ and associated outcomes $z_i \in Z$ before the set of available actions $a_i \in A$ [12]. The probabilities of the possible states of nature are affected by the outcomes of the experiments. New probabilities are updated via Bayes Rule (3) and thus obtaining posterior probabilities:

$$p''[\theta_i \mid z_k] = \frac{p[z_k \mid \theta_i] p'[\theta_i]}{\sum_j p[z_k \mid \theta_j] p'[\theta_j])} \tag{3}$$

Here, $p''[\theta_i|z_i]$ is the posterior probability of state θ_i given experiment outcome z_i, $p[z_i|\theta_i]$ is the conditional probability of outcome z_i given state of nature θ_i and $p'[\theta_i]$ is the prior probability of state of nature θ_i.

In simple terms, an a-posteriori analysis is limited to the updating of probabilities from the extra information from the outcomes of an experiment whereas pre-posteriori analyses include the analysis of which experiment, if any, is better based on the expected utility of each experiment. For this objective, the value of information (VoI) of a given experiment e_i is computed as:

$$VoI[e_i] = E[e_i] - E[e_0] \tag{4}$$

where $E[e_i]$ is the expected utility of experiment e_i and $E[e_0]$ is the expected value of not implementing any experiment. In the case of decision analysis for structural health monitoring, implementing an experiment it means to implement a specific SHM system. Therefore, as the Value of Information (VoI) increases, there is a corresponding increase in the benefit derived from installing a Structural Health Monitoring (SHM) system..

Combining the concepts of SHM and Bayesian Decision Theory, a tree with the architecture shown in Figure 3 is built. The first branching of the decision tree is the implementation of an SHM damage classifier with positive and negative indications of damage as outcomes.

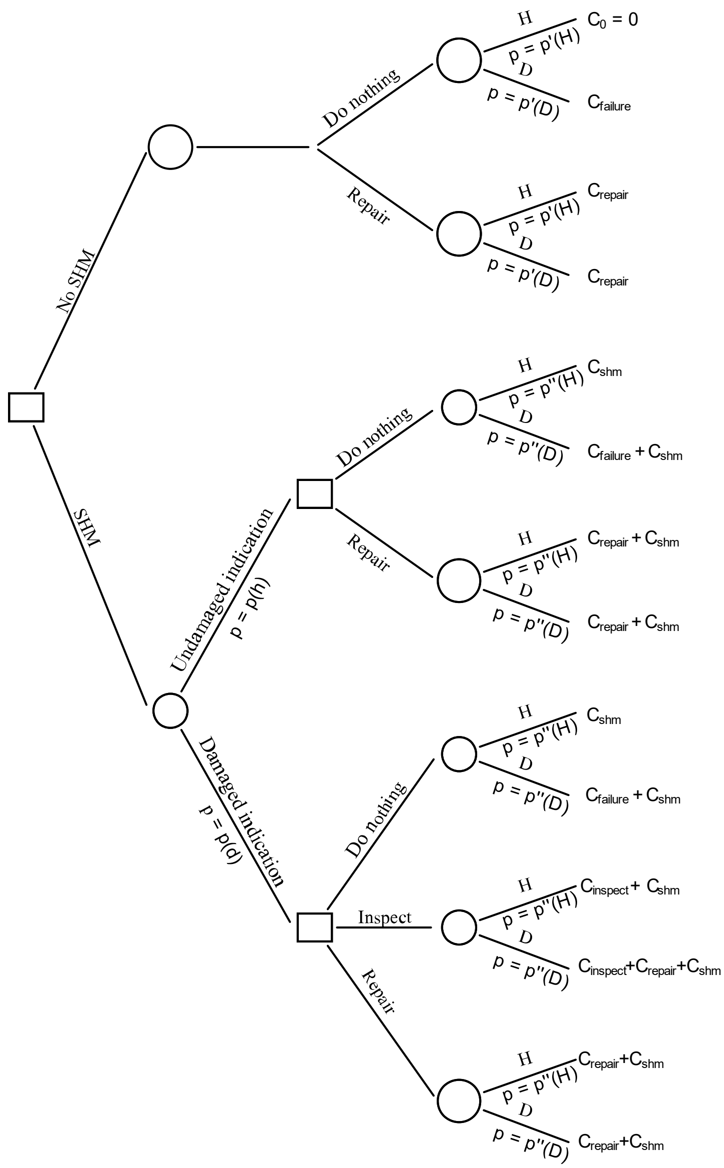


Figure 3. SHM decision tree architecture

The outcomes of the damage classifier have associated probabilities that are dependent on FPR, FNR, and the prior probability of damage in the structure.

$$\begin{aligned} p(h) &= (1-FPR)\cdot(1-p'(D)) + FNR\cdot p'(D) \\ p(d) &= FPR\cdot(1-p'(D)) + (1-FNR)\cdot p'(D) \end{aligned} \tag{5}$$

p(h) and p(d) are the probabilities of negative and positive indications of damage, respectively. TNR is the true negative ratio and FPR is the false positive ratio. p'(D) is the prior probability of damage in the structure, p'(H) = 1-p'(D) is the prior probability of an undamaged structure.

The set of actions A in the decision model is to do nothing, to repair the structure, and to make an additional inspection in case the SHM damage classifier shows a positive indication of damage. Here, it is assumed that inspections and repairs are ideal, meaning that inspections provide complete certainty about the actual condition of the structure, and repairs restore the structure to its original, undamaged state.

A simplified cost model (6) is adopted for the analysis presented in this paper in which the influence of the discount rate is disregarded due to uncertainties regarding the time frame to consider. Additionally, the cost associated to no failure C_0, which would be added to all the possible branches of the tree, is assigned a null value. The states of the structure, undamaged or damaged, and their combinations with the actions yield the costs shown in the tree in Figure 3.

$$C_i(a_i,\theta_i) = C_0 + C_{failure} + C_{repair} + C_{shm} + C_{inspection} \tag{6}$$

Finally, the posterior probabilities of the structure states are calculated and depend on the outcomes and the properties of the SHM system [4]. The posterior probabilities are calculated as follows:

$$\begin{gathered} p''[D\,|\,(h,d)] = \frac{[(1-FNR)^d \cdot FNR^h]\cdot p'(D)}{[(1-FNR)^d \cdot FNR^h]\cdot p'(D) + [FPR^d \cdot (1-FPR)^h]\cdot p'(H)} \\ p''[H\,|\,(h,d)] = 1 - p''[D\,|\,(h,d)] \end{gathered} \tag{7}$$

p’’[D|(h,d)] is the posterior probability of a damaged structure given observations (h,d), p’’[H|(h,d)] is the posterior probability of an undamaged structure given indications (h,d), h is the number of times the damage classifier yields a negative indication of damage, i.e., undamaged, and d is the number of times the damage classifier yields a positive indication of damage, i.e., damaged.

Sensitivity Analysis

The VoI of the SHM system for different repair/SHM and failure/SHM cost ratios is calculated. The ratios range from 1 to 40 for the repair/SHM cost ratio and from 1 to 200 for the failure/SHM cost ratio. Two different cases, with different prior probabilities of damage p’(D), and positive and negative indications of damage, d and h, are analysed. The numerical values used as input for the decision model are presented in table II.

Case 1, where the prior probability of damage is close to zero, corresponds to a typical case of a newly built or retrofitted structure. In this case, additionally, the damage classifier yields positive indications of damage implying damage in the structure.

Case 2, where the prior probability of damage is high, corresponds to a structure which is thought to be damaged as in the case of accidental loading. In this case, the damage classifier yields negative indications of damage implying an undamaged structure.

TABLE II. FIXED VALUES AND CASES FOR SENSITIVITY ANALYSIS.

	FPR (%)	FNR (%)	p' (D)	h	d
Case 1	6.3	4.1	1.00E-06	1	7
Case 2	6.3	4.1	0.20	7	1

Number of sensors and expected utilities

Subsets from the complete sensor network shown in Figure 2a are selected to observe the relationship between the expected costs associated with the SHM system and the number of sensors in the system. The subset selection is based on its proximity to the damage locations shown in Figure 2b and c. The error rates for these smaller SHM systems, which are calculated following the same procedure as for the complete SHM system [13], are shown in table III.

The main interest of this analysis is to visualize the influence of the number of sensors on the expected costs associated with the SHM system Thus, the repair/SHM and failure/SHM cost ratios are set to 10 and 100, respectively. The prior probability of damage p'(D) is fixed to 10^{-6}. Furthermore, the negative and positive indications of damage, h and d, are 1 and 10, respectively.

RESULTS

Sensitivity Analysis

Figure 4 shows the results from the sensitivity analysis. The contour plots present the VoI for different repair/SHM and failure/SHM cost ratios for the FPR, FNR, prior probability of damage, and negative and positive indication of damage values indicated in table II.

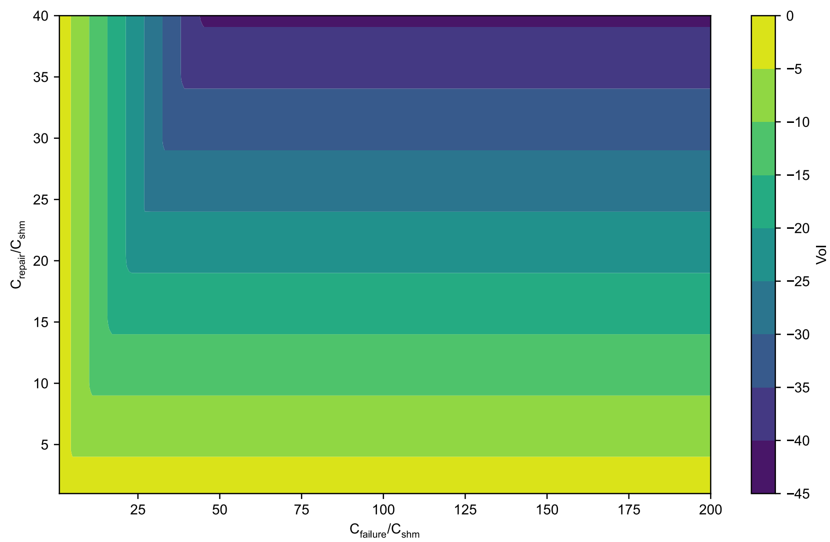
The results of Case 1, shown in Figure 4a, indicate that the VoI is not positive for the considered range of cost ratios. The highest possible VoI for Case 1 is zero for structures with low repair/SHM and low failure/SHM cost ratios

TABLE III. FIXED VALUES AND INDICATIONS FOR THE SENSITIVITY ANALYSIS.

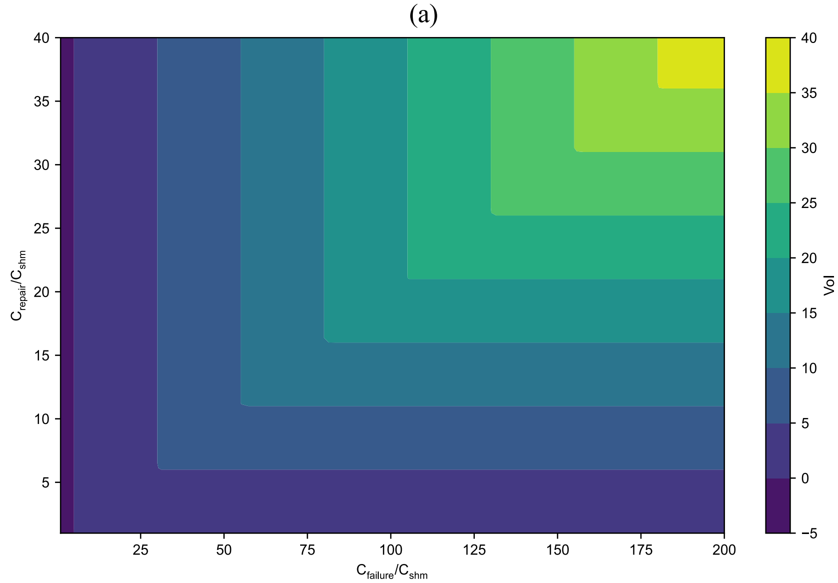
Number of Sensors	FPR (%)	FNR (%)
1	0	73.4
5	0	40.5
10	1.3	32.2
15	1.3	20.9
20	1.3	15.8
25	1.3	13.4
30	2.5	11.1
35	3.8	5.6
40	6.3	4.1

Furthermore, the results for Case 1 show that the VoI is more sensitive to the failure/SHM ratio. For any repair/SHM cost ratio over 5, the VoI diminishes rapidly as the failure/SHM cost ratio reaches a critical value after which the VoI does not change. For example, if the repair/SHM cost ratio is 20 the VoI would start at 0 and reach -20 for a failure/SHM cost ratio of 25, then the VoI keeps this same value of -20 for the rest of the failure/SHM cost ratio range. For any failure/SHM cost ratio over 50, the VoI is inversely proportional to the repair/SHM cost ratio.

The results of Case 2, shown in Figure 4b, indicate that the VoI is positive for almost the complete considered range of cost ratios. The lowest possible VoI for Case 1 occurs for structures with very low failure/SHM cost ratios. The VoI seems equally sensitive to both cost ratios.



(a)



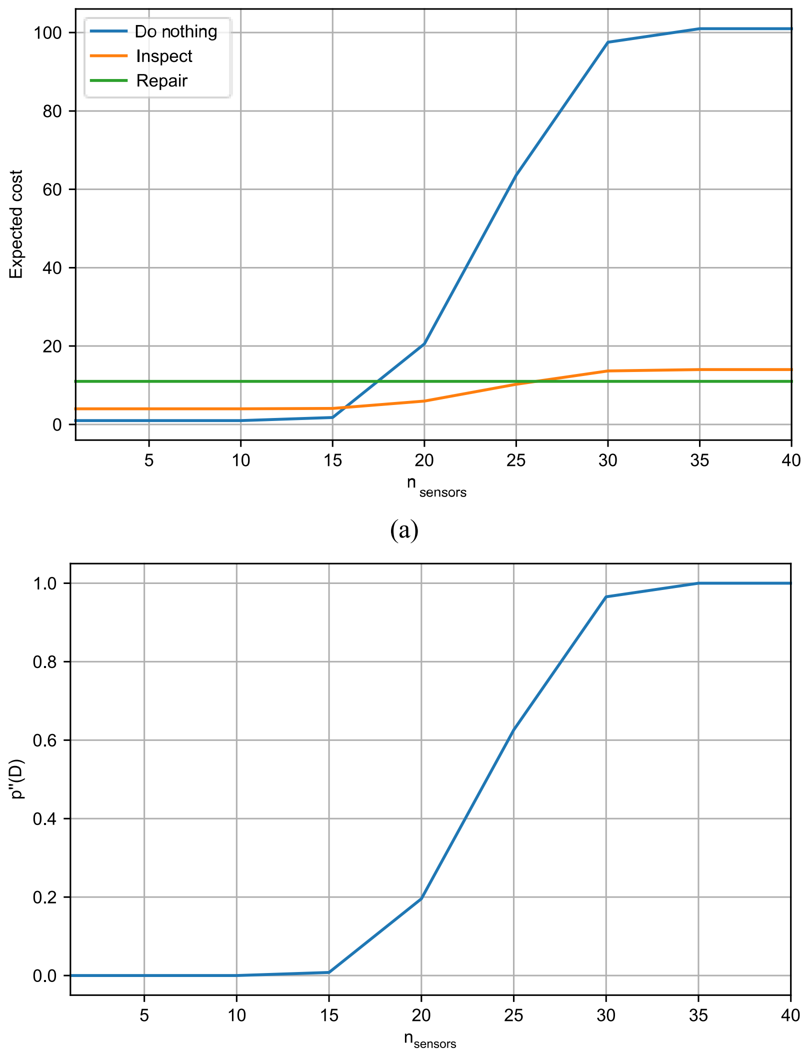
(b)

Figure 4. Sensitivity analysis results (a) Case 1 (b) Case 2

Number of sensors and expected utilities

The influence of the number of sensors of the SHM system on the decision model properties is presented in Figure 5. The FPR and FNR corresponding to the number of sensors for these calculations are presented in in table III.

Figure 5a shows the expected costs of each action from the decision model in the case of installing an SHM system for different number of sensors of the SHM system. The expected cost of doing nothing has an S-shape curve and it increases rapidly when the number of sensors is larger than 15. The expected cost of inspecting rises slightly as the number of sensors increases whereas the expected cost of repairing is invariant. Figure 5a indicates that the least expensive action is to do nothing up to 15 sensors. To inspect the structure is the least expensive action between 15 and 25 sensors. Then, for numbers of sensors larger than 25, to repair the bridge is the least expensive action.



(a)

(b)

Figure 5. Influence of the number of sensors on decision model properties (a) Number of sensors vs. Expected costs (b) Number of sensors vs. posterior probability of damage.

Figure 5b presents the posterior probability of damage for different number of sensors, and it also is an S-shapes curve. Even though the SHM system yields 10 indications of damage the posterior probability of damage is close to 10^{-6}, the prior probability of damage, for fewer than 15 sensors. As more sensors than 15 are used, the posterior probability of damage rapidly reaches the maximum value.

DISCUSSION

Sensitivity Analysis

Case 1 represents newly built or retrofitted structure. The prior probability of damage is close to zero and the damage classifier yields positive indications of damage. In this case the VoI is always non positive, and it reduces as the costs of failure and repair increase. This means that implementing an SHM system would not be convenient for the decision model. The expected utilities of implementing an SHM system compared to not implementing it are equal even for low failure/SHM and repair/SHM cost ratios, where the VoI is zero.

The results for Case 1 indicate that the expected utilities decrease when more information is supplied regarding a structure that is thought to be undamaged but evidence suggests otherwise. The prior-undamaged structure is monitored and the structure appears to be damaged since the damage classifier yields positive indications of damage but the expected utilities associated to this new knowledge are counterintuitively negative. These results, shown in Figure 4a, reflect a limitation of the decision model related to the probability of damage when no SHM system is applied. The expected costs of SHM grow because the posterior probability of damage increases and multiplies the cost of failure. The VoI reduces linearly as the SHM expected costs increase because the expected cost of not implementing the SHM system is constant since the prior probability of damage is not changed. This limitation could be addressed by selecting a decision model that considers the prior probability of damage of the structure without an SHM system as a function of time.

Case 2 represents a structure believed to be damaged. The prior probability of damage is high and the damage classifier yields negative indications of damage. In this case the VoI is positive in most of the cost ratio combinations, and it increases as the costs of failure and repair increase. This means that implementing an SHM system would be convenient for the decision for structures with failure/SHM and repair/SHM cost ratios higher than 25 and 5, respectively.

The results for Case 2 indicate that the expected utilities increase when more information is supplied regarding a structure that is thought to be damaged but evidence suggests otherwise. The increasing VoI of the SHM, shown in Figure 4b, reflects the increase in expected utility when SHM backs up the certainty regarding an undamaged state of the structure. The VoI, in this case, is proportional to the importance of the structure since it increases as the failure/SHM and repair/SHM cost ratios increase, and high ratios are indications of a large and important structure.

Number of sensors and expected utilities

The results from this analysis show the influence the increase in the posterior probability of damage has on the VoI of SHM in more detail. In these calculations the prior probability of damage is close to zero and the damage classifier yields positive indications of damage, such as in Case 1 of the sensitivity analysis. Figure 5a provides the expected utilities of each action from the model in the case of installing an SHM system and indicates the least expensive action. It can be observed from Figure 5a that the expected cost of doing nothing is an s-shaped curve and increases rapidly once a critical number of sensors is passed. This is because the expected cost of doing nothing is the product of the posterior probability of damage times the cost of failure. The posterior probability of damage, shown in Figure 5b, also increases rapidly as the number of sensors reaches the same critical value.

The rapid increase in the posterior probability of damage can be explained by the fact that the system is more sensitive as more sensors are used. In other words, the probability of damage is higher for a sensor network of 30 sensors than for a sensor network of 10 sensors when considering 10 indications of damage as shown in Figure 5b. This means that the posterior probability of damage of the decision model depends on the sensitivity of the implemented SHM system.

Conclusion

This paper explores the effects that different properties of an SHM system have on the decision-making processes using Bayesian decision analysis. Several simplifications are considered to observe simple but insightful relations. SHM properties are kept constant to obtain a basic comprehension of the relationship between VoI and cost ratios. Additionally, costs are kept constant and the effect of SHM variables on the expected costs of actions is computed.

The results show that the implementation of an SHM system has the effect of reducing the uncertainty associated with the true state of the structure. In the common case where the prior probability of damage is small and the SHM damage classifier yields several positive indications of damage, hinting towards a damaged structure, the expected cost associated with implementing the SHM system increase, and its corresponding VoI decreases. This limitation could be overcome by including time dependencies in the prior probability of damage. In the case where the prior probability of damage is large and the SHM damage classifier yields several negative indications of damage, hinting towards an undamaged structure, the expected cost associated with implementing the SHM system decrease, and its corresponding VoI increases. In conclusion, the applicability of utilizing SHM results for informing and updating damage probabilities in decision models is demonstrated.

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