# Bridge Frequencies Identification from the Dynamic Response of a Passing Vehicle

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**ABSTRACT**

Based on the coupling nature of a vehicle moving on a bridge, Yang et al [1] proposed an indirect bridge frequency approach where the bridge response can be recorded by the passing vehicle. Then, the bridge frequencies can be obtained from the dynamic response of the moving test vehicle. Following this concept, in this study, an experimental setup was designed for a test vehicle moving on a bridge to measure the bridge frequency. From the present experimental results, the indirect bridge inspection method is feasible for monitoring the dynamic characteristics of a bridge.

**INTRODUCTION**

Conventional bridge structural health monitoring (SHM) involves installing a large number of sensors directly on a bridge, which is costly and labour-intensive. To simplify the bridge monitoring procedure in practice, Yang et al [2] proposed a vehicle-bridge interaction (VBI) model to extract beam frequencies from the response of a passing sprung mass unit. Based on this theoretical background, Yang and colleagues [2-11], presented a series of experimental and theoretical investigations on an indirect measurement method. On the experimental side, [3] used a two-wheel cart towed by a light truck to scan the fundamental vibration frequency of a suspension bridge and confirmed the feasibility of the indirect measurement method for monitoring bridge frequencies. This approach is referred to as the indirect monitoring method.

Based on the indirect method, [12] used an instrumented moving vehicle to extract the fundamental frequency of a simple short-span bridge. It was found that the response of the test vehicle moving at speed v over a bridge of span length $L$ is dominated by frequencies such as vehicle frequency $f_v$, bridge frequency $f_b$ and driving frequency ( $\pi v / L$ ), and that lower moving speeds allow the vehicle to extract the bridge dynamic properties with higher spectral resolution and less noise from road surface roughness [13]. The tire properties of a hand-drawn test vehicle and the transfer of bridge dynamic properties to the moving test apparatus were investigated by Yang et al [8].

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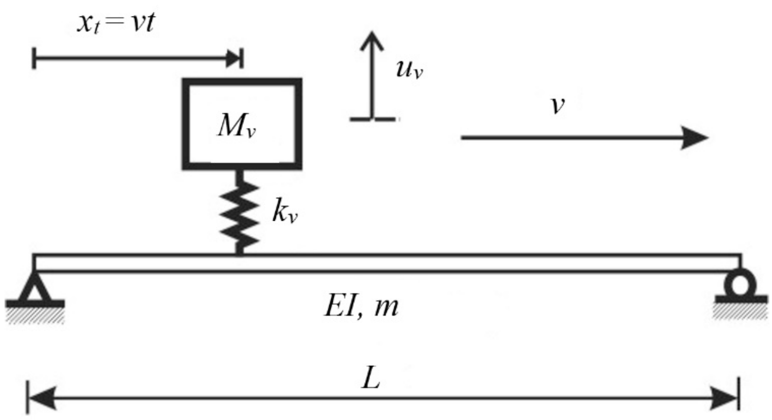



Figure 1. Schematic diagram of a sprung mass moving on a beam.

The global filtering method was used for damage detection based on the operational deflection shape curvature extracted from the vehicle response by Zhang et al [14]. The change in dynamic properties of a bridge was detected by comparing the patterns of dynamic parameters in a scaled laboratory test by Kim et al [15].

As a test vehicle travels on a bridge, the passing vehicle can be considered as an active actuator to excite the bridge and also as a response receiver to capture the vibration data of the bridge on which it moves. With this concept, an experimental setup for measuring the bridge frequency is carried out to verify the indirect method. From the present experimental results, the indirect method is verified to be an efficient and mobile assessment technique suitable for bridge structural health monitoring.

## RESPONSE OF A VEHICLE RUNNING ON A SIMPLE BEAM

Let us consider the simplified model shown in Fig. 1 for a sprung mass moving on a simple beam with smooth pavement. [16]. For the case of a moving load across the span of a beam, which is transient in nature, the response of the beam can be well simulated by considering only the fundamental mode of vibration [17]

In Fig. 1, the following parameters are adopted for the beam: $m$ = mass per unit length, $c$ = damping, $L$ = span length, $EI$ = flexural rigidity, and the following for the sprung mass unit: $v$ = moving speed, $M_v$, = lumped mass, and $k_v$ = spring stiffness. We can write the equations of motion for the beam and the sprung mass moving over the beam as [1]:

$$m\ddot{u} + c\dot{u} + EIu'''' = -\left(p_0 - M_v\ddot{u}_v\right)\delta\left(x - vt\right) \quad 0 \le t \le L/v \tag{1}$$

$$M_v\ddot{u}_v + k_v u_v = k_v u(x_t, t) \tag{2}$$

where $(\bullet)' = \partial(\bullet)/\partial x$, $(\dot{\bullet}) = \partial(\bullet)/\partial t$, $u(x,t)$ = vertical deflection of the beam, $u_v$ = vertical displacement of the sprung mass, $p_0 = M_v$g = weight of the sprung mass, g = gravity acceleration, $L$ = span length, $\delta(\bullet)$ = Dirac's delta function, and $x_t = vt$ = the position of the moving sprung mass on the beam. For a simply supported beam, the following boundary conditions are adopted:

$$u(0,t)=u(L,t)=0,\;\; EIu''(0,t)=EIu''(L,t)=0 \tag{3}$$

From the viewpoint of practical bridges, the mass of a bridge *mL* is usually much larger than that of a running vehicle $M_v$, i.e. $M_v/mL \ll 1$ [18], As shown in reference [6], the numerical studies demonstrated that once the mass ratio of a coach to a simply supported bridge is smaller than 0.05, the dynamic effect of moving vehicles moving on the bridge could be neglected, which is the case studied in this paper. So the inertial force ( $M_v\ddot{u}_v$ ) in Eq. (1) can be neglected and the deflection $u(x,t)$ of the beam subjected to a moving static force can be approximated as [1]

$$u(x,t)\simeq\sum_{n=1}\Delta_s\left[\sin\frac{\pi vt}{L}-S_1\sin\omega_b t\right]\times\sin\frac{\pi x}{L} \tag{4}$$

in which the speed parameter $S_1$ is defined as

$$\Delta_{s1}=\frac{-2p_0L^3}{EI\pi^4\left(1-S_1^2\right)} \tag{5}$$

$$S_1=\frac{\pi v}{\omega_b L} \tag{6}$$

$$\omega_b=\left(\frac{\pi}{L}\right)^2\sqrt{\frac{EI}{m}} \tag{7}$$

With the beam response shown in Eq. (4), one can obtain the displacement response of the sprung mass as

$$\begin{aligned}u_v(t)=&\frac{\Delta_{s1}}{\left(1-S_v^2\right)}\left[\frac{1}{2}\left(1-\cos\frac{2\pi vt}{L}\right)-S_v^2\sin^2\frac{\omega_v t}{2}\right]\\&+\frac{\Delta_{s1}S_1}{2}\left[C_{b1}\cos\left(\omega_b t-\frac{\pi vt}{L}\right)+C_{b2}\cos\left(\omega_b t+\frac{\pi vt}{L}\right)+C_v\cos(\omega_v t)\right]\end{aligned} \tag{8}$$

where

$$S_v=\frac{2\pi v}{\omega_v L},C_{b1}=\frac{-1}{1-\left(\frac{\omega_b}{\omega_v}-\frac{S_v}{2}\right)^2},\; C_{b2}=\frac{1}{1-\left(\frac{\omega_b}{\omega_v}+\frac{S_v}{2}\right)^2},\; C_v=-C_{b2}-C_{b1} \tag{9a-d}$$

As shown in Eq. (8), the vehicle response contains three important components: vehicle frequency ( $\omega_v$ ), bridge frequency ( $\omega_b$ ) and driving frequency ( $2\pi v/L$ ). Let us consider the case that the moving speed of the test vehicle is restricted in very low speeds (< 2m/s), then one can find the approximation of $\omega_b \pm \pi v / L \approx \omega_b$, from which Eq. (8) can be approximated as

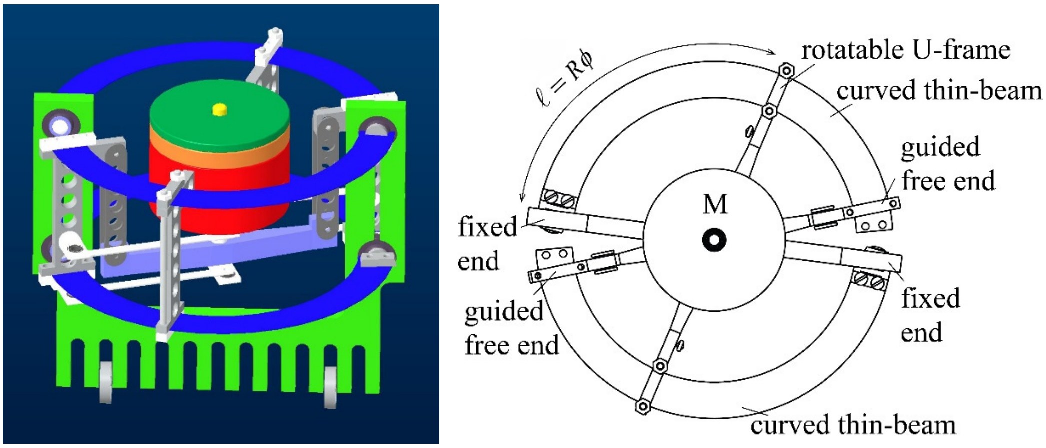
$$u_v(t) \approx \frac{\Delta_{s1}}{\left(1-S_v^2\right)}\left[\frac{1}{2}\left(1-\cos\frac{2\pi vt}{L}\right)-S_v^2\sin^2\frac{\omega_v t}{2}\right]-\frac{\Delta_{s1}S_v^2}{2}\left[1-\left(\omega_b/\omega_v\right)^2\right]^2\cos\left(\omega_b t\right) \quad (10)$$

So the dominant frequency $\omega_b$ of the test beam would exist in Eq. (10) and one can detect the natural frequency of the test beam using the passing test vehicle. For this consideration, the flexible footbridges would not be considered using the present approach. Therefore, the indirect bridge monitoring method is useful to perform bridge health monitoring. In the following section, a VBI experimental setup will be introduced and used to verify the feasibility of the indirect method.

## DESIGN OF AN SDF TEST VEHICLE WITH ADJUSTABLE STIFFNESS

The frequency of the test vehicle is one of the key factors influencing the acceleration response of the test vehicle moving on the beam. Following the idea of indirect frequency measurement using a passing SDF test vehicle presented earlier, an SDF sprung mass unit with adjustable frequencies in the laboratory was designed as a response receiver moving on the beam.

As shown in the sketch in Figure 2, the SDF test vehicle consists of a lumped mass ($M$) supported by two sets of semi-circular double beams, one end of which is fixed to the supported U-frame and the other has a vertically guided free end. The guided free ends of the curved double beams are rigidly connected to the U-frame. The central U-frame is allowed to rotate along the semi-circular beams to vary the support stiffness, so that the vertical vibration frequency can be adjusted by changing the subtended angle $\phi$ of the SDF sprung mass system.




(a) (b)

Figure 2. A 3D sketch model and top view of the test vehicle.

(a) 3D sketch model of the SDF test vehicle;

(b) semi-circular double-beam set and supported mass ($M$).

By changing the subtended angle ($\phi$) or arc length ($\ell = R\phi$) from the fixed end of the curved beam, the corresponding vertical frequencies of the test vehicle can be measured. Table I shows the natural frequencies measured at different subtended angles. As shown in Table I, the vertical frequency of the test vehicle decreases as the subtended angle ($\phi$) increases, because the larger the subtended angle, the smaller the stiffness to support the lumped mass. This means that the present test vehicle can be used to determine the bridge frequencies with different natural frequencies.

TABLE I. MEASURED FREQUENCIES OF THE TEST VEHICLE WITH DIFFERENT SUBTENDED ANGLES

| $\phi$ (deg.) | 90 | 105 | 135 |
|---|---|---|---|
| $f_v$ (Hz) | 10.86 | 8.67 | 6.41 |

## EXPERIMENTAL SETUP OF INDIRECT METHOD FOR BEAM FREQUENCIES

The indirect measurement of an SDF instrumented vehicle moving on a test beam is carried out in laboratory experiments. The laboratory experiments were carried out by driving the test vehicle on the 4 m steel beam (see Figure 3). In the U-shaped steel beam with a total mass of 33.3 kg, the accelerometers were installed in the quarter and centre spans of the beam. The test beam is supported at both ends by partial torsional restraints. The first natural frequency of the beam is 7.02 Hz.

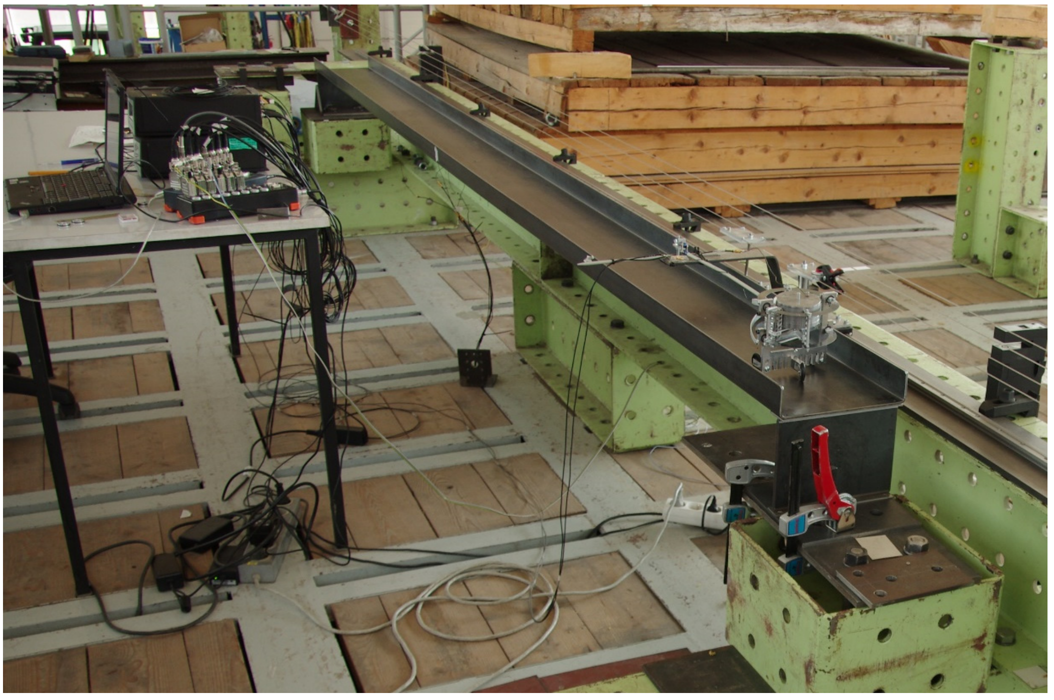


Figure 3 Experimental test beam.

To measure the vertical response of the test vehicle, a Brüel-Kjaer type 4374 piezoelectric accelerometer was attached to the lumped mass of the instrumented test vehicle. The signals from the accelerometers were recorded at a sampling rate of 1000 Hz and transmitted to a computer via charge amplifiers. In order to drive the test vehicle on the test beam at a constant speed, a set of guides was designed to control the speed by means of a speed converter in a drive motor.

## EXPERIMENTAL PROCEDURE AND RESULTS

The measurement equipment includes accelerometers, data acquisition and response analysis software for the moving test vehicle. The response of the moving test vehicle was measured during its passage over the beam to verify the dynamic parameters. Considering the smooth characteristics of the road surface of the measured steel beam, the roughness of the steel beam is neglected in the present study. The moving speed of the test vehicle is chosen to be 0.2 m/s.

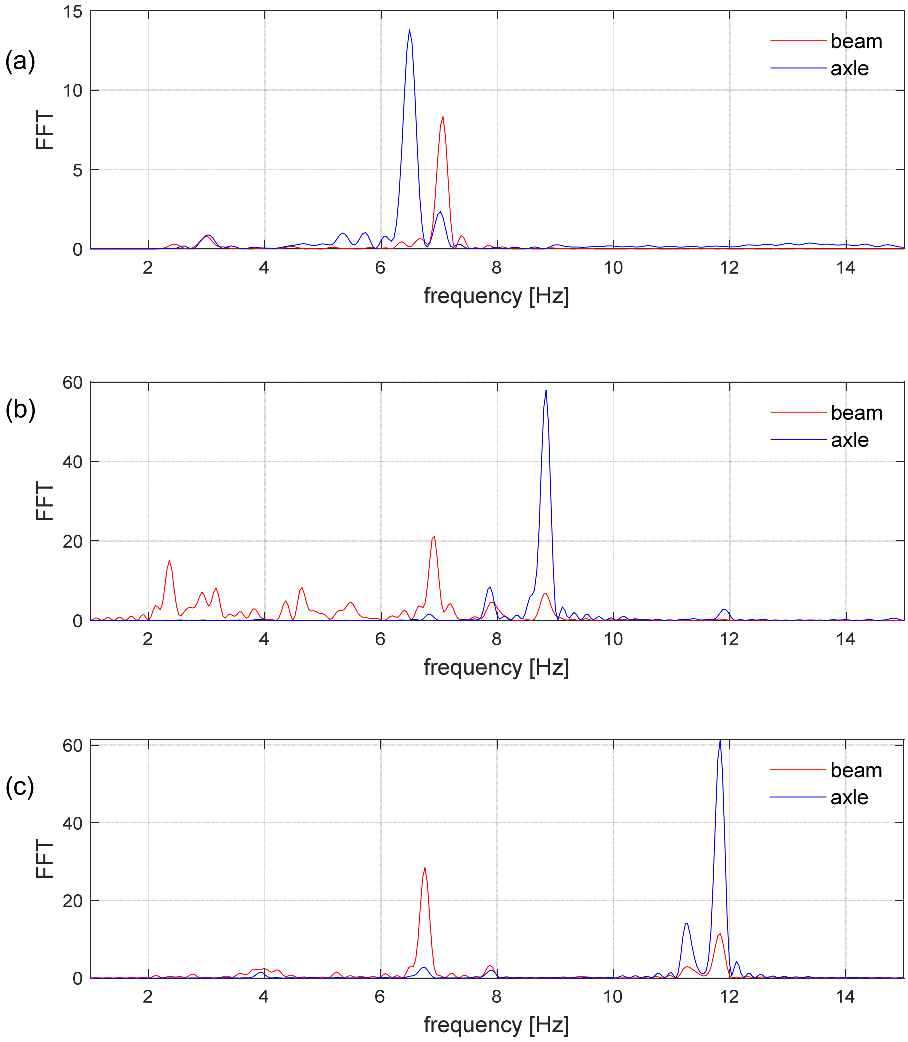



Figure 4. Experimental results for beam frequency analysis with various springs.

(a) $\phi = 135^\circ$; (b) $\phi = 105^\circ$; (c) $\phi = 90^\circ$.

As can be seen from Figure 4 the solutions obtained by the experiments show first natural frequency accordance between the beam and vehicle response. But it was a problem to find agreement at a higher frequency, which was caused by the high damping coefficient of the beam model.

## SUMMARY

This paper represents a preliminary experimental study on the feasibility of detecting the fundamental bridge frequency from the dynamic response of a vehicle passing over the bridge. From both the analytical and experimental studies, it is found out that the bridge frequency is included in and can be extracted from the vehicle acceleration spectrum**.** The results obtained from the tests were carried out in the laboratory ITAM AS CR shows the potential applications of using the proposed indirect method to identify the dynamic characteristics of the bridge, e.g. the natural frequency. Moreover, the present experimental results have verified the feasibility of using a passing test vehicle to detect dynamic information of a bridge, which can be regarded as a preliminary study using the indirect frequency measurement for bridges. Future research studies should be continued to address the experimental works, which are not covered in this preliminary study, including different roughness of pavement, damping and suspension mechanisms of the vehicle, measurements in situ, and so on.

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