

# Scour Damage Detection of Bridge Piers Using the Vibration-Based Method: Numerical Study on a Steel-Concrete Bridge in France

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## ABSTRACT

The scour is one of the main reasons for bridge collapse. Soil removal around bridge foundations alters their embedment conditions and affects the stability of the structure. Therefore, scour detection is crucial to ensure the safety of bridges and prevent their failures. This paper presents a vibration-based analysis method for detecting scour in its early stages, using frequency values as a scour indicator. Scour is represented by the degradation of foundation stiffness. A robust expression is developed to calculate the variation of frequency as a function of stiffness variations. Different scour scenarios are numerically simulated on a steel-concrete bridge model, revealing that frequencies related to horizontally displaced modes are sensitive to scour, while the deck bending modes are insensitive. The SVD method used to solve the inverse problem has proven to be robust in estimating stiffness reduction based on frequency changes and identifying the location of scour.

## 1. INTRODUCTION

Scour is the erosion of soil around bridges piers and other hydraulic structures. This phenomenon arises especially during floods, where the strong water flow, altered by the presence of a construction, carries sediments away [1-3]. Due to scour, the lateral resistance of the structure and the shear force in the columns are reduced. Scour can also increase the shear force and bending moment values in the pile which can increase the possibility of its failure [4].

Indeed, scour is recognized as one of the main causes of bridges collapses and damages worldwide [5-6]. For example, in the United States, 53% of the 500 bridges that collapsed between 1998 and 2000 were caused by scour [5].

In order to mitigate scour effects, several methods including visual inspection, geophysical and acoustic methods have been developed in the past years

for scour detection. However, each of these techniques has its own limitations [7].

Although, visual inspection is the most straightforward method, it is limited by the accessibility of the inspection site and the expertise of the inspectors [8]. In addition, geophysical methods such as sonar and echo sounder, or acoustic methods like ground penetration radar require difficult and expensive under water implementation that can be damaged during floods [9-10]. Additionally, their accuracy depends on many factors such as water turbulence, soil properties and the equipment used.

Recently, researchers have proposed a new method for scour detection based on the vibration analysis of bridge structures. This method tends to determine the presence of scour by observing the changes in the dynamic response of the structure induced by the scour, rather than surveying the riverbed. It is a non-invasive technique that offers a continuous real-time monitoring of the bridge and that is feasible even during floods, when the scour risk is at its highest. [11-12] studied the effect of the scour depth on the first frequency of a deep foundation. The experiments conducted on a small scale and a full-scale pile, where the scour is modelled as an incremental soil removal, show that the natural frequency decreases gradually as the scour depth increases. [13] conducted an experimental test on a steel bridge to assess foundation scour by increasing the free length of the piles. They found that frequencies related to horizontally-displaced modes decrease with increasing scour depth, while frequencies related to vertical modes were insensitive to scour. The first frequency was reduced by up to 6% with a scour depth equivalent to 18% of the original pile length. According to authors [11-17], tracking changes in natural frequencies has potential to remotely detect piers scour.

This paper uses an indirect consequential method that involves detecting scour based on the bridge's dynamic response. The rest of this paper is organized as follow: In section 2, the methodology including the theoretical formulations of the frequencies sensitivity and the inverse problem is described. In section 3, the numerical model of a real bridge is introduced. Section 4 presents a series of noteworthy results obtained from different scour scenarios. And, the final section concludes the study and outlines potential avenues for future research.

## 2. METHODOLOGY

Scour is considered as a degradation of foundation rigidity. The natural frequency is selected as the indicator parameter for scour detection in this study. The small perturbations method with an approximation of the first order is applied to establish the sensitivity expression of the eigenvalues concerning the foundation rigidity variation. This expression is then employed to construct the sensitivity matrix that relies only on modes sensitive to scour. The objective of this study is to compute the loss of rigidity based on measured frequencies values. To achieve this, an inverse problem is formulated, and the solutions to the inverse problem are obtained by applying the singular value decomposition (SVD) method to the sensitivity matrix.

## Theoretical formulas

The bridge is considered as an assembly of Euler-Bernoulli beams that are perfectly connected to each other. The soil-structure interaction along the foundation surface is represented by a rotational spring with an adequate stiffness.

The eigen value problem of this structure can be written in continuum and a discretized form at the ends of the piles as follow:

$$\int_{\Omega} \underline{\underline{\varepsilon}}(\underline{U}) : \underline{\underline{E}} : \underline{\underline{\varepsilon}}(\underline{V}) d\Omega + \sum_{j=1}^n C_j \theta_j(\underline{U}) \theta_j(\underline{V}) = \omega^2 \int_{\Omega} \rho \underline{U} \cdot \underline{V} d\Omega \quad (1)$$

The angular frequency and the displacement vector ( $\omega_i, \underline{U}_i$ ) are the solutions that make the equation (1) true for every virtual displacement vector  $\underline{V}$  where:

- $\underline{\underline{\varepsilon}}(\underline{U})$  is a second order small strain tensor
- $\underline{\underline{E}}$  is a fourth order elasticity tensor
- $\theta_j$  is the rotation at the end of a pile. It is linearly dependent from  $\underline{U}$ .
- $C_j$  is the rotational stiffness of the foundation where ‘j’ is the index that refers to the ‘j’ spring and ‘n’ is the total number of springs.
- $\rho$  is the density of the structure and  $\Omega$  is its volume.

To write this problem in a general way, the bilinear symmetric forms are used.

$$a(\underline{U}, \underline{V}) = \omega^2 m(\underline{U}, \underline{V}) \quad (2)$$

Where  $\underline{U}$  and  $\omega$  are the solution of the eigen value problem  $\nabla \underline{V}$ . And,  $a(\underline{U}, \underline{V})$  is the bilinear symmetric form associated to the elastic deformation energy of the structure. In this case, it is formed by 2 terms: the first one written in continuum mechanics forms refers to the flexural behavior of the structure. The second term is expressed in a discretized form and it describes the elastic energy of the springs placed on the ends points of the piles.

$$a(\underline{U}, \underline{V}) = \int_{\Omega} \underline{\underline{\varepsilon}}(\underline{U}) : \underline{\underline{E}} : \underline{\underline{\varepsilon}}(\underline{V}) d\Omega + \sum_{j=1}^n C_j \theta_j(\underline{U}) \theta_j(\underline{V}) \quad (3)$$

$m(\underline{U}, \underline{V})$  is the bilinear symmetric form related to the kinetic energy.

$$m(\underline{U}, \underline{V}) = \int_{\Omega} \rho \underline{U} \cdot \underline{V} d\Omega \quad (4)$$

By introducing small elastic perturbations, the form  $a$  is perturbated by  $\delta a$ . Consequentially, the eigen value  $\omega_i^2$  corresponding to the ‘i’ mode of vibration is perturbated by  $\delta \omega_i^2$  and the mode shape  $\underline{U}_i$  by  $\delta \underline{U}_i$ .

$$a(\underline{U}_i + \delta \underline{U}_i, \underline{V}) + \delta a(\underline{U}_i + \delta \underline{U}_i, \underline{V}) = (\omega_i^2 + \delta \omega_i^2) m(\underline{U}_i + \delta \underline{U}_i, \underline{V}) \quad (5)$$

After using the symmetrical property of the forms, a and m, simplifying (5) by the equation (2) and neglecting the terms of the second order, it is written:

$$\delta a(\underline{U}_i, \underline{V}) = \delta \omega_i^2 m(\underline{U}_i, \underline{V}) \quad (6)$$

$\delta a(\underline{U}_i, \underline{V})$  is also a bilinear symmetric form. In the case of the scour, the stiffness of the foundations is degraded. To represent this damage, small perturbations of the first order are introduced to the rotational stiffness of the foundations ( $\delta C_j$ ). The expression of  $\delta a(\underline{U}_i, \underline{V})$  is written as follow:

$$\delta a(\underline{U}_i, \underline{V}) = \sum_{j=1}^n \delta C_j \theta_j(\underline{U}_i) \theta_j(\underline{V}) \quad (7)$$

By taking  $V = U_i$ , and normalizing the modes to the mass  $\int_{\Omega} \rho \underline{U}_i^2 d\Omega = 1$ , the sensitivity expression of the eigen value to the stiffness variations is established.

$$\delta \omega_i^2 = \frac{\delta a(\underline{U}_i, \underline{U}_i)}{m(\underline{U}_i, \underline{U}_i)} = \sum_{j=1}^n \theta_j(\underline{U}_i)^2 \delta C_j = \sum_{j=1}^n b_{ij}^2 \delta C_j \quad (8)$$

The rotations  $\theta_j$  are linearly dependent from the displacement vector  $\underline{U}_i$ . In Euler-Bernoulli beam, rotation is the derivative of the displacement. The expression of the rotation  $\theta_j(\underline{U}_i)$  in the curvilinear coordinate system is:  $\theta_j(\underline{U}_i) = \underline{U}_i, s$ .

For  $j$  going from 1 to  $p$ ,  $C_j$  refers to the rotational stiffness in the first principal transverse direction. And for  $j$  going from  $p+1$  to  $n=2p$ , it refers to the stiffness in the second principle direction, noting that  $p$  is the number of piles.

## Inverse problem

In order to solve the inverse problem, which means determining the foundations stiffness variations ( $\delta C_j$ ) from the measured or numerically obtained frequencies changes ( $\delta \omega_i^2$ ), the singular value decomposition (SVD) method is applied to the sensitivity matrix. The SVD factorizes the matrix A of size  $(m \times p)$  into 3 matrices U, S and V such that:

$$A_{(m \times p)} = U_{(m \times m)} S_{(m \times p)} V^T_{(p \times p)} \quad (9)$$

Where U is an  $(m \times m)$  orthogonal matrix, S is an  $(m \times p)$  diagonal matrix with non-negative entries called singular values, and V is an  $(p \times p)$  orthogonal matrix [18]. The SVD method is useful for solving inverse problems because it allows to compute the pseudoinverse of the matrix A, which is a generalization of the inverse that can be applied to non-square matrices. The stiffness variations vector is calculated by multiplying the pseudoinverse of the matrix of sensitivity A by the vector of the given frequencies.

### 3. NUMERICAL APPLICATION

#### Bridge description

The bridge is a steel-concrete bridge crossing the Loire river in France. It has a total length of 601 m and is composed of 8 spans, supported by 7 piers and 2 abutments. The deck is a mixed section that comprises a 30 cm thick concrete slab of 10.75 m wide fixed on two I-section steel girders. Pot bearing devices are installed between the superstructure and the piers, allowing the sliding in-plane movement of the structure. The piers of the bridge are constructed from concrete and have a circular section with a diameter of 3.5 m. Each pier is supported by a concrete foundation that has a rectangular section of 9x6 m<sup>2</sup> and a height ranging between 2 and 4 m. It is noteworthy that only 4 of the 7 piers are constructed in the water. The following work focuses on the part of the bridge located over the water.

#### Finite Element model

The numerical model of the bridge was created using CESAR, a finite element software. The bridge was simplified into a frame structure constructed in a 3D space (Figure 1). The mixed deck section was homogenized into concrete and represented by a 1D beam element with specific properties, including  $EI = 1.72 \times 10^{11}$  N.m<sup>2</sup> and  $EA = 1.6 \times 10^{11}$  N. The foundation stiffness was represented by a rotational spring located at the end of each pier. The initial stiffness in both transverse directions to the axis of the pier were evaluated using a 3D linear elastic model of the concrete foundation and the soil with a Young's modulus of 250 MPa. The obtained values are within the range of  $10^{10}$  N.m/rad. The bearings between the deck and piers were considered rigid in this model, allowing for in-plane rotations and longitudinal sliding displacement, except for the second pier, which was fixed perfectly to the deck. The boundary conditions at the two abutments of the bridge were represented by simple supports that allowed the sliding displacement.

#### Scour scenarios

Table I. presents a summary of the numerically simulated scour scenarios, indicating the amplitude of the scour (percentage of stiffness loss per single foundation), its location and its direction (X: direction of water flow, Y: direction of traffic). These simulations aid in identifying the vibration modes that are sensitive to scour and enable the computation of the sensitivity coefficients in equation (4).

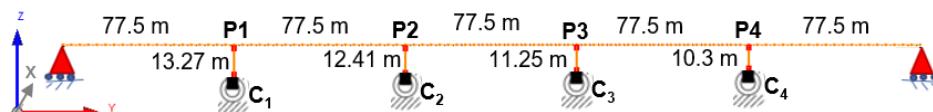


Figure 1. Simplified model of the bridge

## 4. RESULTS

### Modal analysis

The vibration modes of the bridge were calculated by the numerical model within the frequency band width of 0-15Hz. The frequencies obtained from each global scour simulation were compared to those of the reference simulation with no scour. Figure 2(a). illustrates the relative change in frequency for each mode compared to the reference frequency value. The results show that 10 of the 40 studied modes are sensitive to changes in foundation stiffness. For an overall scour equivalent to a 10% stiffness loss in both directions at all piles, the maximum frequency is reduced by 2.6%. The vibration modes of the sensitive frequencies can be classified into two categories: the out-of-plane vibration modes (modes 7, 8, 9, 10, 16 and 19) that are simulated at low frequency range see Figure 2(b), and the in-plane vibration modes of the piers (modes 3, 22, 32 and 33). Except for pier P2, whose vibration is coupled with the translation movement of the deck due to the rigid link between them, the three other piers vibrate independently at high frequencies. The frequencies corresponding to the flexion modes of the deck are insensitive to scour. According to Figure 2(a), the identified sensitive mode can tell the direction of the scour hole. For example, in the case where rigidity losses were made only in the direction of the water flow (blue line), only the out-of-plane modes were affected. Furthermore, the vibration modes of the piers are only sensitive to scour that occurs along the bridge's longitudinal axis, as shown by the red line of Figure 2(a).

TABLE I. SCOUR SCENARIOS AROUND THE BRIDGE MODEL

| Model        | Simulation name | Relative stiffness loss | Scour location | Scour direction |
|--------------|-----------------|-------------------------|----------------|-----------------|
| Reference    | NS1R0           | 0 % - no scour          | -              | -               |
| Global scour | NS2GS10         | 10 %                    | P1, P2, P3, P4 | X, Y            |
|              | NS3GS5X         | 5 %                     | P1, P2, P3, P4 | X               |
|              | NS4GS5Y         | 5 %                     | P1, P2, P3, P4 | Y               |

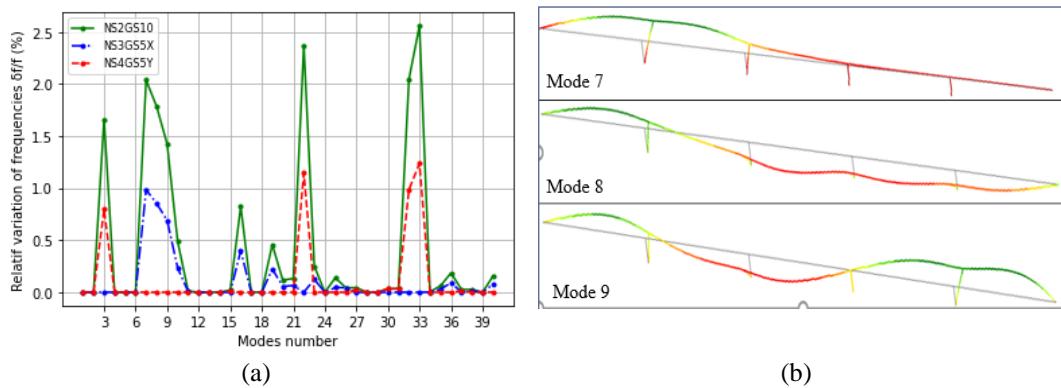


Figure 2. (a) Relative variations of the frequencies due to foundations stiffness loss. (b) First three out-plane modes sensitive to the scour

## Sensitivity matrix

By applying the frequency sensitivity expression to the numerical model of the bridge, the sensitivity matrix is built. In the following application, the stiffness variation in both global directions X and Y are assumed to be equal. Using the relative variations, the spectral perturbation is given by:

$$\frac{\delta f_i}{f_i} = \frac{1}{2 \lambda_i} \left[ b_{i1} \frac{\delta C_1}{C_1} + b_{i2} \frac{\delta C_2}{C_2} + b_{i3} \frac{\delta C_3}{C_3} + b_{i4} \frac{\delta C_4}{C_4} \right] \quad (10)$$

Knowing that  $\lambda_i = \omega_i^2 = (2\pi f_i)^2$  and  $b_{ik}$  is the sensitivity matrix coefficient with  $i$  the number of the mode and  $k$  the number of the pile ( $k=1$  to 4 in the case of the studied bridge). The expression of  $b_{ik}$  is written in local coordinates system of the pile.

$$b_{ik} = \left[ C_{zk} \left( \frac{dU_{zi}}{dx} \right)_{x=end\ Pier\ k}^2 + C_{yk} \left( \frac{dU_{yi}}{dx} \right)_{x=end\ Pier\ k}^2 \right] \quad (11)$$

The sensitivity matrix  $[A]$  obtained is formed by 4 columns referring to the number of piles and 10 rows representing the 10 sensitive modes to the scour detected in the previous section. The rotations values in  $z$  and  $y$  directions are obtained from the reference numerical simulation at the points situated at the end of the piers for every sensitive mode. The conditioning number of  $A$  is equal to 1.66. This means that the problem is well posed and the errors won't be amplified. The variation of stiffness at the piers can be directly calculated from the in-plane modes 3, 22, 32 and 33. But, if the out-plane modes were only measured, the sensitivity matrix is rectangular and non-inversible. In order to calculate the rotational stiffness variation using the changes in the frequencies, the SVD method is used.

$$\begin{pmatrix} \delta f_3/f_3 \\ \delta f_7/f_7 \\ \delta f_8/f_8 \\ \delta f_9/f_9 \\ \delta f_{10}/f_{10} \\ \delta f_{16}/f_{16} \\ \delta f_{19}/f_{19} \\ \delta f_{22}/f_{22} \\ \delta f_{32}/f_{32} \\ \delta f_{33}/f_{33} \end{pmatrix} = \begin{bmatrix} 0 & 1.53 \times 10^{-1} & 0 & 0 \\ 1.53 \times 10^{-1} & 3.54 \times 10^{-2} & 1.11 \times 10^{-3} & 8.95 \times 10^{-5} \\ 7.57 \times 10^{-3} & 3.3 \times 10^{-2} & 6.22 \times 10^{-2} & 6.07 \times 10^{-2} \\ 4.64 \times 10^{-3} & 3.89 \times 10^{-2} & 3.34 \times 10^{-5} & 9.01 \times 10^{-2} \\ 8.72 \times 10^{-5} & 4.75 \times 10^{-3} & 3.41 \times 10^{-2} & 5.78 \times 10^{-3} \\ 1.25 \times 10^{-2} & 2.97 \times 10^{-2} & 2.57 \times 10^{-2} & 9.14 \times 10^{-3} \\ 6.35 \times 10^{-3} & 1.35 \times 10^{-3} & 1.21 \times 10^{-2} & 2.34 \times 10^{-2} \\ 2.21 \times 10^{-1} & 0 & 0 & 0 \\ 0 & 0 & 1.9 \times 10^{-1} & 0 \\ 0 & 0 & 0 & 2.4 \times 10^{-1} \end{bmatrix} \begin{pmatrix} \frac{\delta C_1}{C_1} \\ \frac{\delta C_2}{C_2} \\ \frac{\delta C_3}{C_3} \\ \frac{\delta C_4}{C_4} \end{pmatrix} \quad (12)$$

## Inverse Problem

To test the efficiency of this method, several scour cases are simulated numerically on a modified model of the bridge where the geometrical or material properties are slightly changed. The original reference model was modified by reducing the Young's modulus of the deck by 2% and increasing the piers diameter

by 1%. Different scour cases of one or multiple piers were simulated on the modified model. For each simulation, a theoretical reduction value of the foundation stiffness was applied to the springs and the frequencies values were extracted. The use of the relative changes of these perturbed frequencies for the inverse problem of stiffness variations estimation prevents the inverse crime and tests the robustness of the method. The simulated values of the relative stiffness reduction at the end of each pier and the calculated values of these stiffness reductions obtained by solving the inverse problem using the SVD method are reported in TABLE II for various scour scenarios. As we can see, the method is robust and was able to estimate the stiffness variation of the foundations even while using the frequency variations of a perturbated model. The maximum relative error committed for estimating a theoretical  $\delta C/C$  equals to 10% is 4.8%. This error appears to be lower for smaller values of stiffness degradations: a relative error of 2 % and 1% was captured respectively for a theoretical  $\delta C/C$  equals to 5 % and 2 %. These results show that the calculated stiffness error is due to the 1<sup>st</sup> order approximation.

## 5. CONCLUSION

In conclusion, this study has demonstrated the potential of using the dynamic response of a bridge to detect scour at the foundation level. A numerical simplified model of a steel-concrete bridge was created and several scour cases were simulated by modelling the scour as a certain percentage loss of the foundation stiffness. By evaluating the changes of frequency values, it was found that the bridge's vibration modes can be divided into two categories: the vertical displacement modes which are insensitive to scour, and the horizontally displaced in-plane and out-of-plane modes which are both sensitive to scour. The in-plane modes correspond to the vibration of the piles and can detect the degradation of the foundation stiffness only if the scour occurs along the longitudinal axis of the bridge, while the out-of-plane modes are sensitive to scour formed in the direction of water flow. Moreover, the stiffness reduction values are simply calculated from the sensitivity expression depending only on sensitive modes. Solving the inverse problem using SVD method was found to be a robust method for estimating the rigidity loss with very low error percentages. However, it is possible that not all 10 sensitive modes identified in this theoretical study can be measured in real life, resulting in an ill-posed problem. In such cases, it becomes necessary to employ regularization techniques to address the issue.

TABLE II. CALCULATED VALUES OF STIFFNESS VARIATIONS

| Simulations<br>Names | Simulated $\delta C/C$ (%) |           |           |           | Calculated $\delta C/C$ (%) using SVD |              |              |              |
|----------------------|----------------------------|-----------|-----------|-----------|---------------------------------------|--------------|--------------|--------------|
|                      | P1                         | P2        | P3        | P4        | P1                                    | P2           | P3           | P4           |
| MNS1                 | <b>10</b>                  | 0         | 0         | 0         | <b>10.43</b>                          | 0.0004       | -0.003       | -0.0006      |
| MNS2                 | 0                          | <b>10</b> | 0         | 0         | 0.04                                  | <b>10.28</b> | 0.04         | -0.02        |
| MNS3                 | 0                          | 0         | <b>10</b> | 0         | 0.002                                 | 0.02         | <b>10.48</b> | 0.02         |
| MNS4                 | 0                          | 0         | 0         | <b>10</b> | 0.005                                 | -0.004       | 0.1          | <b>10.42</b> |
| MNS5                 | 0                          | <b>10</b> | <b>5</b>  | 0         | 0.05                                  | <b>10.28</b> | <b>5.07</b>  | -0.02        |
| MNS6                 | <b>10</b>                  | <b>5</b>  | <b>5</b>  | <b>2</b>  | <b>10.42</b>                          | <b>4.9</b>   | <b>5.03</b>  | <b>1.98</b>  |
| MNS7                 | <b>10</b>                  | <b>10</b> | <b>10</b> | <b>10</b> | <b>10.42</b>                          | <b>10.22</b> | <b>10.47</b> | <b>10.45</b> |

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